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Essays on Asset Pricing Factor Models: Evidences on Idiosyncratic Volatility, Emerging Markets and Monetary Policy

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação em Administração de Empresas of the Departamento de Administração, PUC-Rio as partial fullfilment of the requirements for the degree of Doutor em Administração de Empresas.

Advisor: Prof. Marcelo Cabús Klotlze

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Abstract

Leite, André Luís; Klotzle, Marcelo Cabús (Advisor). **Essays on Asset Pricing Factor Models: Evidences on Idiosyncratic Volatility, Emerging Markets and Monetary Policy.** Rio de Janeiro, 2018. 97p. Tese de Doutorado – Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

Since its proposition in the 1960s, the capital asset pricing model and its expansions, in particular the modeling proposed by Fama and French between the years 1992 and 2015, caused an enthusiastic debate about the economic interpretation of its factors. It has been demonstrated in the academic literature that variables describing the set of future investment opportunities should command a risk premium and should be correlated with the Fama and French factors. Another issue that has always been discussed is the application of this type of modeling to emerging markets. Weaker and less structured economies would follow the same rationality of developed markets? Fama-French's expansions add to the CAPM model factors that represent size, value, operating profitability, and corporate investment policy in two basic model versions. The first, proposed in 1993, adds to the excess market return a factor of size and a factor of value. It is usually called the three-factor model. The second, proposed in 2015, adds to the three-factor version a factor of operational profitability and a factor of companies' investment policy. It is usually called the five-factor model. With the use of these models and the financial concepts involved, this thesis studies the possibility that the innovations in the average market variance, decomposed into two factors, one representing the average market variation and another representing the average market correlation, could increase the explanatory capacity of the three-factor model with respect to the excess returns of stock portfolios. It also studies the ability of the five-factor model to best explain stock portfolio returns in emerging market economic blocks relative to the original CAPM and the three-factor model. Finally, the study shows that innovations in the inflation index and innovations in the slope of the interest curve are proxies for size, value, profitability, and investment factors, and, together with excess market returns, explains cross-section of excess returns on stock portfolios better than the five-factor model.

Keywords

Idiosyncratic Volatility; Expected Returns; Asset Pricing; Price of Risk; Emerging Markets; State Variables; Fama and French

Resumo

Leite, André Luís; Klotzle, Marcelo Cabús. **Ensaios sobre Modelos de Fatores para Apreçamento de Ativos: Evidências sobre Volatilidade Idiossincrática, Mercados Emergentes e Política Monetária.** Rio de Janeiro, 2018. 97p. Tese de Doutorado – Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

Desde sua proposição, na decada de 60, o modelo de apreçamento de ativos de capital e suas expansões, em particular a modelagem proposta por Fama e French entre os anos de 1992 e 2015, causou um entusiasmado debate sobre a interpretação econômica de seus fatores. Foi demonstrado na literatura acadêmica que variaveis que descrevem o conjunto das futuras oportunidades de investimento devem comandar um prêmio de risco e deveriam ser correlacionadas com os fatores de Fama e French. Uma outra questão sempre discutida é a aplicação desse tipo de modelagem à mercados emergentes. Economias mais fracas e menos estruturadas seguiriam a mesma racionalidade de mercados desenvolvidos? As expansões de Fama-French acrescentam ao modelo do CAPM fatores que representam o tamanho, o valor, a lucratividade operacional e a politica de investimento das empresas, em duas versões básicas de modelo. A primeira, proposta em 1993, acrescenta ao excesso de retorno de mercado um fator de tamanho e um fator de valor. É normalmente chamada de modelo de três fatores. A segunda, proposta em 2015, acrescenta a versão de três fatores um fator de lucratividade operacional e um fator de politica de investimentos das empresas. É normalmente chamada de modelo de cinco fatores. Com o uso desses modelos e dos conceitos financeiros envolvidos, esta tese estuda a possibilidade de que as inovações na variância média do mercado, decomposta em dois fatores, um representando a variação média do mercado e outro representando a correlação média do mercado, pudesse aumentar a capacidade explicativa do modelo de três fatores no que se refere aos excessos de retornos de portfólios de ações. Ela também estuda a capacidade do modelo de cinco fatores de melhor explicar o retornos dos portfolios de ações, em blocos econômicos de mercados emergentes, em relação ao CAPM original e ao modelo de três fatores. Finalmente, o estudo mostra que as inovações no indice de inflação e as inovações da inclinação da curva de juros são *proxies* para os fatores de tamanho, valor, lucratividade e investimento, e, em conjunto com o excesso de retorno do mercado, conseguem explicar o *cross-section* dos excessos de retornos dos portfólios de ações melhor do que o modelo de cinco fatores.

Palavras-chave

Volatilidade Idiossincrática; Retornos Esperados; Preços de Ativos; Preço de Risco; Mercados Emergentes; Variáveis de Estado; Fama e French

Summary

Let me tell you something you already know. The world ain't all sunshine and rainbows. It's a very mean and nasty place, and I don't care how tough you are, it will beat you to your knees and keep you there permanently if you let it. You, me, or nobody is gonna hit as hard as life. But it ain't about how hard you hit. It's about how hard you can get hit and keep moving forward; how much you can take and keep moving forward. That's how winning is done!

Sylvester Stallone, as Rocky, in "*Rocky Balboa*". 2006

1
Introduction

Since the emergence of portfolio theory – proposed by Markowitz (1952), bringing together the concepts of efficient/inefficient resource allocation, risk/return, and diversification – different asset pricing models have had their origins inspired by the assumptions presented then. Among these, the Capital Asset Pricing Model (CAPM) became the most famous and the reference for academic studies.

CAPM was independently and almost simultaneously proposed by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). The Sharpe version became the most well-known, resulting in the author receiving the Nobel Prize in 1990. Despite different empirical problems, CAPM is quite popular today, due to its simplicity and intuitiveness. The model establishes a relationship between the return on an asset, the return free of risk, and average market return. Those are Fundamental Factor Models, that is, they use observable asset specific fundamentals such as company size, market capitalization and book value, for example, to construct common factors that explain portfolios' excess returns. Since its proposition in the 1960s, the capital asset pricing model and its expansions, in particular the modeling proposed by Fama and French between the years 1992 and 2015, caused an enthusiastic debate about the empirical validity of the model and the economic interpretation of its factors.

The first chapter of this work expand Chen and Petkova (2012) work for Brazilian market. Ang, Hodrick, Xing, and Zhang (2006) study the relation between market returns' volatility and portfolio excess returns. They find a puzzling result. Although the volatility is priced as a risk factor, portfolios with high (low) volatility exhibited a lower (higher) expected return, contradicting the theory. Chen and Petkova (2006) propose to break the market volatility up into two components: Average Volatility (AV) and Average Correlation (AC), and test their innovations separately as pricing factors. Decomposition of market variance is carried out in a way that the product of the two components corresponds to total volatility. They find the average variance component better predicts the effects of a worsening or an improvement in the investment environment than total variance, as well as commanding a negative price of risk for expected return on the portfolios. The average covariance component is not significant in both cases. The economic explanation presented by Chen and Petkova (2012) for this anomaly is the fact that participants in the US stock market perceive investment in R&D in the companies as a risk-reducing factor, or rather, as positive volatility, i.e. originating from a factor (R&D) that increases the value of the company in periods of uncertainty. The present work test this methodology for Brazilian data and find that in Brazil, these anomalous future returns are not repeated; economic agents require a positive premium on future returns for portfolios with higher volatility. In fact, research in R&D in Brazil is almost inexistent, so the effect seen at US companies do not repeat for local economy. Without mitigating risk effect of R&D, Brazilian's portfolios are perceived as, in fact, more risky, and for this reason, have a higher discount rate on their cash flows in the case of an increase in average volatility.

In the second chapter, RMW and CMA – Fama and French (2015) operational profitability factor and investment factor, respectively – are calculated for 12 emerging markets countries and three emerging markets economic blocks. With these factor an analyses is conducted to compare the ability of CAPM and Fama and French three- and five-factor model in pricing portfolios returns ordered by, size and value, size and operational profitability and size and investment. We seek to verify if our results are close to those found for developed markets. We deal with many problems that affect the results from such samples, including small sample sizes, low portfolio diversification, political instability, economic problems, and border hazard, among others. Despite these issues, we find results that, if not as strong and clear as those for developed markets, show some evidence that they follow the same trends found in US data and other developed countries.

For the third and last chapter, the relation among macroeconomic variables and Fama and French factors is studied. It has been demonstrated in the academic literature that variables describing the set of future investment opportunities should command a risk premium and should be correlated with the Fama and French factors. Following many interesting discussions, one question is straightforward: what macroeconomic variables could give us some intuitions about the five-factor model of FF (2015)? Petkova (2006) presents a set of four macroeconomic variables that are proxies for HML (value factor described in Fama and French, 1993) and SMB (size factor, described in Fama and French, 1993) factors. We show this set of variables cannot explain the RMW factor. Bernard (1986) demonstrates that underlying firm characteristics could create interaction between unexpected inflation and operating profitability. Inspired by this result, we added innovations to CPI to the set of innovations to economic variables proposed by Petkova (2006) and the RMW factor loses its explanatory capability. More than that, in the presence of the market excess return and the first principal component of the new five-variable set, all other four factors of FF (2015) lose their explanatory ability. Finally, the study shows that innovations in the inflation index and innovations in the slope of the interest curve are proxies for size, value, profitability, and investment factors, and, together with excess market returns, explain cross-section of excess returns on stock portfolios better than the five-factor model.

2
Effects of Idiosyncratic Volatility in Asset Pricing

2.1
Introduction

For a factor model, with factors that reflect the return on tradable portfolios, the constant for the equation that describes the model, normally defined as α, serves as an indicator of how well specified the model is. In the case of omitted factors, α will be different to zero and statistically significant. In the Fama and French (1996) model, in particular, statistical tests indicate the existence of missing factors. In this case, the volatility of residuals, i.e. the idiosyncratic volatility (IV), is influenced in proportion to the sensitivity of a portfolio to the missing factor. Portfolios that are more sensitive to missing factors have a higher IV than less sensitive ones. New factors should thus be included in the model, in order to improve its specification. A way of approaching this problem was presented by Chen and Petkova (2012) and, in this paper, is demonstrated for the Brazilian case.

Asset pricing theory states that idiosyncratic volatility (IV), defined as being the standard deviation of the residuals from the Fama and French (1996) model, should not be priced. On the other hand, Merton (1987) shows that, if investors are not able to correctly diversify their portfolios, then idiosyncratic volatility should be positively rewarded. In short, specific risk in a portfolio should be irrelevant or positively related with the expected return on it.

Ang, Hodrick, Xing, and Zhang (2006) show that volatility of market return is priced as a risk factor in asset portfolios. Based on this evidence, their studies tested this measure as a factor missing in the Fama and French (1996) model. The results were contradictory in relation to the theory that suggests that IV should be irrelevant or positively related with return; portfolios with high (low) IV exhibited a lower (higher) expected return. Chen and Petkova (2012) then presented the proposal of breaking market volatility up, in a search to clarify this result. The methodology suggests breaking market variance up into two components – average variance and average covariance – and testing them separately as factors in the model.

Decomposition of market variance is carried out in a way that the product of the two components corresponds to total volatility. Orthogonal shocks are estimated for these variables, which are used as two additional factors to the Fama and French (1996) model, in order to estimate their coefficients separately. The results found in the literature, concerning the US data, show that the average variance component better predicts the effects of a worsening or an improvement in the investment environment than total variance, as well as commanding a negative price of risk for expected return on the portfolios. The average covariance component is not significant in both cases.

According to Chen and Petkova (2012), citing Merton (1980), it is expected that when average volatility rises, general market volatility also rises, increasing uncertainty, which commands an increase in the expected market risk premium. This should raise companies` discount rate, reducing their value and, consequently, increasing expected return – higher risk, higher return. According to Avramov, Chordia, Jostova, and Philipov (2013), the future returns on US portfolios are negatively related to idiosyncratic volatility and, because of this, form part of a group of returns classified as anomalous. Companies with high IV – in theory, higher risk – exhibit lower return. The economic explanation presented by Chen and Petkova (2012) for this anomaly is the fact that participants in the US stock market perceive investment in R&D in the companies as a risk-reducing factor, or rather, as positive volatility, i.e. originating from a factor (R&D) that increases the value of the company in periods of uncertainty. Thus, the discount rate on cash flows in these companies is increased, but less intensely, which means share values fall less, generating an expectation of proportionally lower return – according to the authors, risk would be lower, therefore return should be lower. In Brazil, these anomalous future returns are not repeated; economic agents require a positive premium on future returns for portfolios with IV. The positive risk premium found in Brazil indicates a different economic perception on the part of participants in the Brazilian stock market. Portfolios sorted by IV in Brazil are not seen as having risk reducing factors, that is, positive volatility is not identified – like that generated by investments in R&D – composing IV in Brazil. Therefore, portfolios are perceived as, in fact, more risky, and for this reason, have a high discount rate on their cash flows in the case of an increase in average volatility, reducing their price and with this increasing the expected return on them; this effect is captured by the positive risk price indicated in the results of this paper. Without the perception of factors that reduce exposure to average volatility (AV), the traditional theory is valid – higher risk, higher return.

The contribution of this paper is empirical in character. The aim is to test, with regards to Brazilian data, a new methodology for pricing financial assets, which presented interesting results for US data. As a result of this, it is believed that it contributes to a better understanding of the issue, and thus presents new evidence regarding portfolio pricing in Brazil.

2.2
Theoretical Framework

Since the emergence of portfolio theory – proposed by Markowitz (1952), bringing together the concepts of efficient/inefficient resource allocation, risk/return, and diversification – different asset pricing models have had their origins inspired by the assumptions presented then. Among these, the Capital Asset Pricing Model (CAPM) became the most famous and the reference for academic studies.

CAPM was independently and almost simultaneously proposed by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). The Sharpe version became the most well-known, resulting in the author receiving the Nobel Prize in 1990. Despite different empirical problems, CAPM is quite popular today, due to its simplicity and intuitiveness. The model establishes a relationship between the return on an asset, the return free of risk, and average market return, in the following way:

$$E(r_{it}) = rf_t + \beta_{im}(rm_t - rf_t) + e_t \quad (1)$$

where r_{it} is the return on asset i in period t; rf_t is the return on the risk-free rate in t; and rm_t is the average market return at the same moment. The β_{im} parameter reflects the sensitivity of the observed asset in relation to variation in market return, i.e. the ratio between asset-market covariance and market variance. The e_t factor represents a pricing "residual" regarding the specific risk of an asset. The standard deviation of e_t is called idiosyncratic volatility, and is cited in the literature as the risk of a particular asset that can be eliminated through diversification.

With a little manipulation in algebra, it is possible to rewrite the above model in the following way:

$$E(R_{it}) = \beta_i(RM_t) + e_t \quad (2)$$

where R_{it} now represents excess return on the asset in question, and RM_t shows excess market return, both in relation to the risk-free rate. The model is then defined, in a very simple way, as a one factor model. Empirical tests for verifying the validity of the model indicate problems; it occurs that, for a great number of assets and/or portfolios, in estimating the coefficients of the equation above, a (α) constant that is different to zero appears with statistical significance. The literature states that, in this type of model, the appearance of a constant that is different to zero indicates possible bad model specification; one or more factors would be lacking that help to explain excess return on assets (Lo & MacKinlay, 2002).

Inspired by these results, different researchers have carried out empirical tests and proposed new ideas in an attempt to eliminate the deviations in the CAPM. To cite some examples: Jensen, Black, and Scholes (1972) carry out CAPM tests and present a two factor model, without risk-free rate loans, which would better represent return on assets; Ross (1977) analyzes the question of market portfolio being efficient in the sense of average variance and, based on this assumption, tests the robustness of the model; Fama and French (1996) analyze five factors that influence return on financial assets and present a three factor model that has become the most popular extended CAPM; MacKinlay (1995) presents a result that suggests that pricing models with various factors do not totally explain deviations in the original CAPM. According to the author, deviations exist that are explained by sources not based on risk.

Among the aforementioned and innumerous other models, the three factor one – presented by Fama and French (1996) – has gained relevance and a sequence of studies and tests, along with the original CAPM. Basically, the authors propose that excesses of returns on financial assets are explained by a model in the following way:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + h_i HML_t + s_i SMB_t + \varepsilon_{it} \quad (3)$$

where R_M, HML, and SMB are the excess return on a market portfolio, the value factor and size factor, respectively, and α_i is the bad model specification indicator. It occurs that, in different empirical tests carried out, the alpha turns out to be different to zero and statistically significant (Chen & Petkova, 2012).

Among different studies regarding the model described above, Ang et al. (2006) showed that market volatility is a risk factor priced in the cross section of shares. Moreover, they argued that a factor missing in the Fama and French (1996) model should influence the idiosyncratic volatility (IV) of a portfolio in proportion to its sensitivity to this factor. Thus, companies with high sensitivity to the missing factor, for example, should show a higher IV, all other things kept equal. Uniting these two concepts, the authors argued that, by sorting shares by IV, they would be able to build portfolios that were priced erroneously by the Fama and French (1996) model, but that could be corrected by included a new factor regarding market volatility. In carrying out the tests, they reached an intriguing and contradictory result: portfolios with higher (lower) IV exhibited lower (higher) expected return, and the spread between portfolios, despite being large, does not explain the difference between future returns. In the search to explain this intriguing result, Chen and Petkova (2012) propose breaking aggregate market variance down into two components – average variance (AV) and average covariance (AC) – and the use of these factors, independently, as factors missing in the model, instead of aggregate market variance. The results show that average variance, as well as being a good predictor for market variance and for the return on portfolios, exhibits a coherent price of risk; this is not verified for average covariance. Moreover, the price of risk found for AV is large and enough to explain the spread between portfolios with high and low IV.

In Brazil, various studies have been carried out in the last decades, with the aim of testing the appropriateness of the model for the domestic market and proposing alterations to improve the results of the original proposal: Costa Jr. and Neves (2000), and Bonomo and Agnol (2003), carried out tests with factors based on specific fundamentals for companies/portfolios and reached conclusions similar to those of the original model; Lucena and Figueiredo (2008) propose new factors to be added to the model based on the parameters ARCH and GARCH. The results presented showed that the factors included turned out to be statistically significant and could be used to improve the Fama and French (1996) model, in Brazil; Rayes, Araújo, and Barbedo (2012) investigate whether a large increase in liquidity in the Bovespa would have affected the ability of the model to explain returns in the Brazilian market. The results suggest that the factors in the model would not explain returns, neither for individual shares nor for portfolios, during the period tested; Mendonça, Klotlze, Pinto, and Montezano (2012) investigate the relationship between idiosyncratic risk and return on shares in Brazil. Following along this line, both an adaptation of the methodology presented by Chen and Petkova (2012) for the Brazilian data, as well as the results found, will be shown below.

2.3
Database and Methodology

2.3.1
The Database

In the elaboration of this study, a list of 352 shares traded on the BM&FBOVESPA between January 2003 and July 2014 was taken as a base. Shares that did not exhibit minimum liquidity, with at least 15 days of trades per month, were then excluded, so that the shares selected had prices that reflected realistic market conditions, at each moment. Shares that exhibited a negative book value were also excluded. These exclusions were carried out monthly, that is, a share excluded in one month could be listed in another month.

In this time period, the Brazilian stock market observed a large number of IPOs, raising the number of liquid shares available every month, primarily between the end of 2006 and the middle of 2008. Due to these conditions, every month there is a different number of liquid shares for building portfolios with regards to the Fama and French (1996) model. The months with a lower or higher number of shares available were, respectively, February 2003, with 43 shares, and February 2014, with 215 shares. On average for the whole period, there are roughly 140 liquid shares per month. Dividing the sample into two parts – before and after the increase of shares on the market – there is, for the first 48 months, an average of 62 liquid shares per month. This first part covers the years from 2003 to 2006; for the second part, covering the period from 2007 to 2014, there are, on average, 182 liquid shares per month.

In the original article by Fama and French (1996), described in item 3.2 (Equation 4) of this paper, in order to generate the model factors, each year at the end of June the companies used in the study are allocated into two groups – big and small (B and S) – according to their market values (share value times the number of shares traded) being below or above the median for the sample. Subsequently, each group is subdivided into three others – low, medium, and high (L, M, and H) – according to the B/M (book-to-market) ratio, which relates company book value to market value. The companies situated among the lowest 30% B/M enter into group L; those among the middle 40% enter into group M; and finally, those among the highest 30% B/M enter into group H. Thus, there are six groups: SL, SM, SH, BL, BM, and BH. The returns on these six portfolios – weighted by the market value of each company – are then generated based on June until June of the following year, when the composition of the portfolios is redefined, using the same methodology described. The SMB factor is generated based on the differences between the average return on the portfolios of small companies (SL, SM, SH) and the average return on the portfolios of big companies (BL, BM, BH). The HML factor is generated based on the differences between the average return on portfolios of companies with high B/M (SH and BH) and the average return on portfolios of companies with low B/M (SL and BL). The information for the calculation of the B/M ratio used for forming the portfolios in year t is observed at the end of tax year of year t-1 (book value) and at the end of December of t-1 (market value). These lags are in order to guarantee that the data is already public in the portfolio assembly data (year t).

To construct the SMB and HML factors, with Brazilian data, the steps described in the original article were followed, introducing only two modifications that were judged to better represent the reality of the Brazilian market. In the original form of the calculation, redefinition of the six portfolios that serve as a base for the factors is carried out annually; in this study, we opted to redefine the portfolios monthly. This was done in order to reflect the large variation in the number of shares traded, as described above. As the factors seek to reflect market conditions at a particular moment, it can be observed that if the redefinition was carried out annually there would be a large distortion in the period between the end of 2006 and the middle of 2008. The second alteration took place in the way of calculating the B/M ratio. In the original form, the data for the portfolios for year t in year t-1 was sought. This was done because the US tax year ends on September 30th. As the Brazilian tax year ends on December 31st, this data was used for book value, and the June 30th value for company market value. This way, both standardization for all the companies as well disclosure of the data on the date of building the portfolios, was guaranteed, with a smaller informational lag than the original, consequently reflecting market conditions closer to the portfolio building data.

For building portfolios sorted by idiosyncratic volatility, each month the shares were sorted by company size and separated into five quintiles. Then, within each quintile, the shares were sorted again by IV and, once again, separated into five quintiles, thus totaling 25 portfolios. The returns weighted by company market value (value-weighted) are the test subjects of this study.

All of the excesses of returns are calculated in relation to the 30 day interest rate, based on the BM&FBOVESPA future interbank deposit data. In order to adjust the interest curve, the Diebold and Li (2006) model is used, with a second curvature factor proposed by Svensson (1994), in the form presented by Almeida, Gomes, Leite, Simonsen, and Vicente (2009).

Both for share value, as well as book value and company market value, the data supplied by Bloomberg was used. The data referring to future ID was obtained in the BM&FBOVESPA's information retrieval system.

2.3.2
The Fama and French Model

The linear relationship that exists between return on assets and risk factors proposed by the authors is described in Equation 4.

$$R_{it} = \alpha_i + \beta_i R_{Mt} + h_i HML_i + s_i SMB_i + \varepsilon_{it} \quad (4)$$

where R_M,, HML, and SMB are the excess return on market portfolio, the value factor and the size factor, respectively, and α_i is the model's bad specification indicator.

In factor models that use excess return or "zero investment" portfolios, if there is an exact relationship between the observed asset and the model factors, then α_i will be zero. The interest here is thus in determining what the relationship is between α_i when it is different to zero, and the error covariance matrix (Σ). To understand this relationship, the optimal orthogonal portfolio (OP) definition described by MacKinlay and Pastor (2000) will be used.

The OP is orthogonal to the other model factors and optimal in the sense that, when included in the model, it forms with the other factors the tangent portfolio. Because it is orthogonal, when included in the model, the OP will preserve the values of β_i, h_i and s_i, as in the original estimation. This way, when inserted into the model, the relationship between returns and the factors becomes:

$$R_{it} = \beta_{poi} R_{pot} + \beta_i R_{Mt} + h_i HML_t + s_i SMB_t + u_{it} \quad (5)$$

where β_{poi} represents the sensitivity of dependent returns in relation to the omitted factor, represented here by the new orthogonal factor. The interaction between this sensitivity and the variance of errors in the original model is obtained by comparing the two equations. Matching the variance of ε_t with the variance of ($\beta_{poi}R_{pot}$ + u_{it}) gives Equation 6.

$$Var[\varepsilon_{it}] = \beta_{poi}^2 Var\left[R_{pot}\right] + Var[u_{it}] \quad (6)$$

It is thus understood that the idiosyncratic volatility of the original model has a positive relationship with the volatility of the omitted factor, in the proportion of asset sensitivity to this factor. The greater the dependent asset's sensitivity to the omitted factor, the greater the idiosyncratic volatility of this asset will be in the original model. It is important to note that, with this configuration, the true idiosyncratic volatility of the asset emerges, $Var[u_{it}]$.

Previous studies – cited in the theoretical framework review – show that, for Brazilian data, there are indications of omitted factors in the Fama and French model (1996).

2.3.3 Omitted Factor

Different studies, such as Campbell (1993), Chen (2003), and Driessen, Maenhout, and Vilkov (2009), suggest that return on assets is correlated to variables that predict return and market variance. Moreover, the temporal series literature suggests that the model's aggregate variance is divided into two components, one related to share variance and the other to covariance.

Inspired by these results, Chen and Petkova (2012) suggest that the factors omitted in the Fama and French (1996) model could be the aggregate market variance components, defined by average variance and average covariance. Thus, they suggest the following model with 5 factors:

$$R_{it} = \alpha_i + \beta_{mi} R_{Mt} + \beta_{HMLi} HML_t + \beta_{SMBi} SMB_t + \beta_{\Delta AVi} \Delta AV_t + \beta_{\Delta ACi} \Delta AC_t + \varepsilon_{it} \quad (7)$$

where R_M, HML, and SMB are the excess return on market portfolio, the value factor, and the size factor, respectively, and ΔAV and ΔAC are the innovations in the aggregate market variance components, calculated as shown below.

The aggregate market variance will be given by:

$$V_t = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_{it}\omega_{jt} Corr\left(R_{it}, R_{jt}\right) sd(R_{it}) sd(R_{jt}) \quad (8)$$

where ω_{it} is the weight of asset i at moment t, applied to calculate the market portfolio weighted by the value of each asset, and N is the total number of assets in the market portfolio. To define the aggregate variance components, the following are defined:

$$AV_t = \sum_{i=1}^{N} \omega_{it} V(R_{it}) \quad (9)$$

as the average variance component, and:

$$AC_t = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_{it}\omega_{jt} Corr(R_{it}, R_{jt}) \quad (10)$$

as the average covariance component. The authors then highlight that, assuming that all the shares have the same individual variance, Equation 8 is reduced to:

$$V_t = AV_t AC_t \quad (11)$$

and the unconditional expectation of equilibrium for return, in the context of discrete-time ICAPM, will be given by:

$$R_{it} = \gamma_0 + \gamma_M \beta_{mi} + \gamma_{HML}\beta_{HMLi} + \gamma_{SMB}\beta_{SMBi} + \gamma_{\Delta AV}\beta_{\Delta AVi} + \gamma_{\Delta AC}\beta_{\Delta ACi} + \varepsilon_{it} \quad (12)$$

where the γs represent the prices of risk related with the market, HML, SMB, AV variation, and AC variation, respectively, and the βs are the factor loads, estimated as shown in Equation 7.

Given that the prices of risk (γs) estimated in Equation 12 refer to the factors, they are equal for all of the portfolios. Thus, different beta sets (factor loads) will lead to different expected returns. Thus, one of the aims of this study is to verify whether portfolios with different IV exhibit loads of different magnitude and/or with opposite signs in relation to the variance components – average variance and average covariance – and whether these loads influence the formation of expected returns on the portfolios.

2.3.4 The Calculation of Market Variance, of AV and of AC

In accordance with French, Schwert, and Stambaugh (1987), (monthly) market volatility and AV were calculated using a correction for the autocorrelation of daily returns. The data used is daily returns within each month. For the aggregate market variance, we have:

$$V_{Mt} = \sum_{d=1}^{D_t} R_{Md}^2 + 2\sum_{d=2}^{D_t} R_{Md}R_{Md-1} \quad (13)$$

where D is the number of days in month t and R_{Md} the market return on day d. For AV, we have:

$$AV_t = \sum_{i=1}^{N_t} \omega_{it}\left[\sum_{d=1}^{D_t} R_{id}^2 + 2\sum_{d=2}^{D_t} R_{id}R_{id-1}\right] \quad (14)$$

where R_{id} is the return on asset i on day d and N_t is the number of assets that exist in month t. The AC component is the value-weighted average of the pairwise correlation of daily returns on each share, each month.

2.3.5 Extracting the Innovations in AV and AC

To evaluate the model in Equation 7 it is necessary to estimate the innovations in AV and AC. For this task the approach described by Campbell (1996), and also assumed by Chen and Petkova (2012), was adopted. A first order VAR was used based on a state vector z_t that contains R_M, HML, SMB, AV, and AC. The model is then described in matrix form by:

$$z_t = Az_{t-1} + u_t \quad (15)$$

where the residuals will be the innovations used as the risk factor in Equation 7. Campbell (1996) explains that it is very difficult to analyze the result of a VAR if the factors are not orthogonalized and normalized in any way. In the above model, the system was triangulated so that the innovations regarding excess market return are not altered, but the rest are orthogonal in relation to those immediately before. Thus, the innovations in AV are orthogonal to those of excess market return, HML and SML. The same occurs for the innovations regarding AC. The system was also normalized so that the innovations of new factors present the same variance; the procedure follows the proposal of Chen and Petkova (2012).

2.4 Results

2.4.1 Main Results

Decomposition of aggregate market variance into two components – average variance and average correlation – was carried out as described in item 3.4. The summarized descriptive statistics are presented in Table 1.

Table 1- Descriptive Statistics

Variance	Mean	Median	Stand. Dev.	Min.	Max.
Vm	0.0061	0.0032	0.0125	0.0005	0.1333
AV	0.0128	0.0090	0.0138	0.0046	0.1332
AC	0.4305	0.4140	0.1458	0.1195	0.9332

Note. This table presents the descriptive market variance statistics, Vm, and its components, average variance, AV, and average correlation, AC. Vm is calculated as described in Equation 13. AV is calculated as described in Equation 14. AC is the value-weighted average of the pairwise correlation of the daily returns in each month.
Source: Developed by the author.

Figure 1.a shows the graphic for market variance and the product of the two estimated components – average variance and average correlation. It is noted that the data series practically overlap, showing that the form adopted for the decomposition of market variance seems quite consistent, despite the equal volatility of all shares hypothesis seeming very strong at first sight. Figures 1.b and 1.c show, separately, graphics of the market variance components – average variance (1.b) and average correlation (1.c).

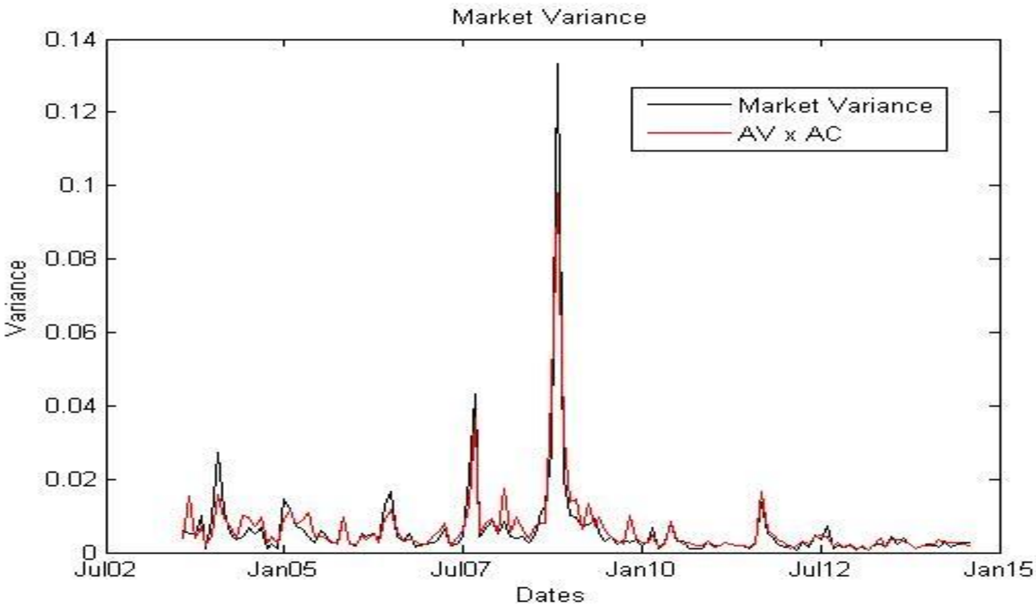

Figure 1.a – Monthly market portfolio variance, Vm, calculated as in Equation 13 and the product of average variance, AV – calculated as in Equation 14, and the average correlation, AC – average of the correlation to the pair of the return on assets each month. Sample period – July 2003 to July 2014.
Source: Developed by the authors.

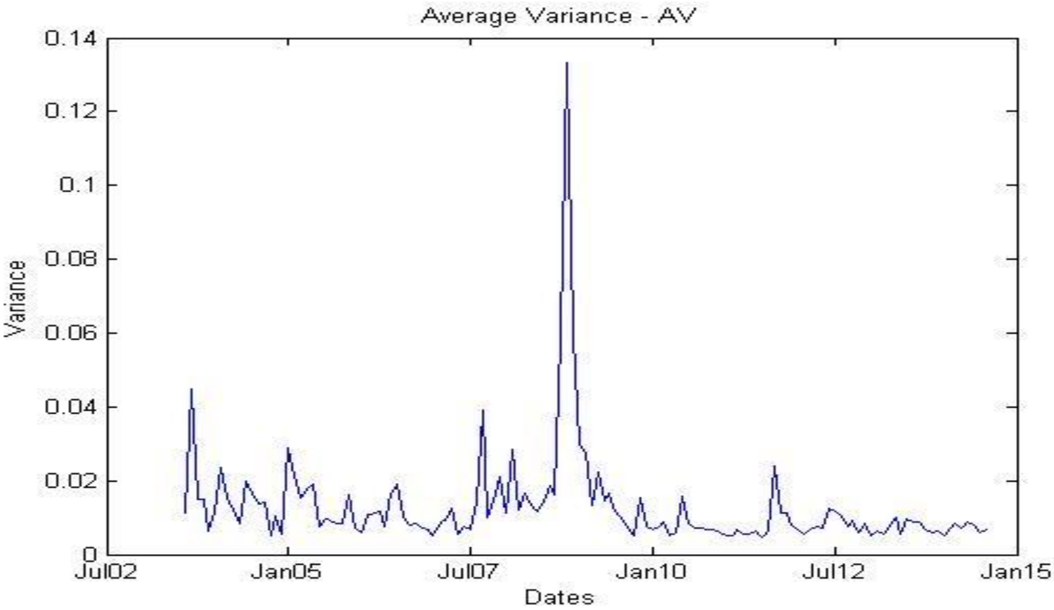

Figure 1.b – Market variance component, average variance, AV – calculated as in Equation 14. Sample period – July 2003 to July 2014.

Source: Developed by the author.

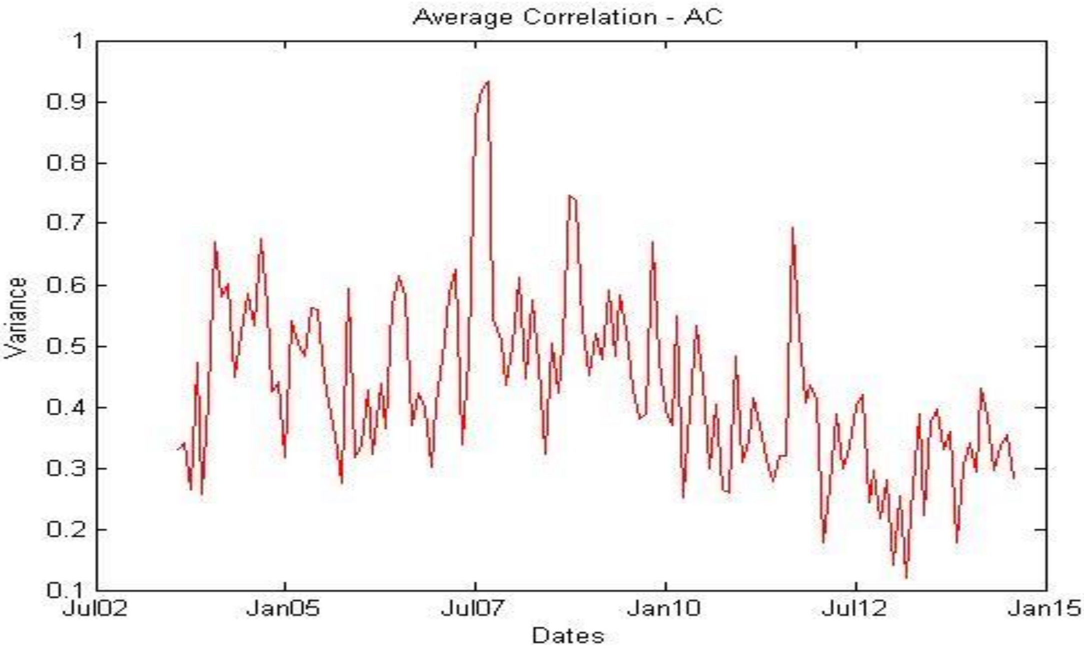

Figure 1.c – Market variance component, average covariance, AC – value weighted average of the correlation to the pair of the daily return on assets each month. Sample period – July 2003 to July 2014,

Source: Developed by the author.

In Table 2 results of the OLS regressions, which seek to analyze the role of average variance and average correlation in explaining changes in market variance and in excess market return, are presented. For all the regressions t statistics from Newey-West were adopted with 4 lags. In column 1 the relationship between market variance and the product of its two components is presented. The constant, despite being statistically significant, exhibits a value very close to zero. The product of the components explains practically all the contemporary variation in market variance, as demonstrated by the R^2 of approximately 90%. In column 2 the relevance of the AV component in relation to changes in market variance, is presented, reaching 81%, while column 3 refers to the average correlation, which captures 21% of these changes. An indication of greater relevance of the average variance component in the behavior of market variance can be noted here. In column 4 the two market variance components in the regression are included. Together, AV and AC explain approximately 82% of contemporary market variance, and only AV turns out to be statistically significant. If compared with the result in column 2, it is perceived that the inclusion of AC in the model does not add practically any explanatory power. In column 5 a test of the predictive ability of AV and AC in the behavior of market variance is carried out, finding an R^2 of 20%; the same test carried out for US data presents an R^2 of 22% (Chen & Petkova, 2012). Column 6 presents a predictive regression of excess market return in relation to average variance and to average correlation of market variance. The R^2 found was 10%, that is, superior to that found in the same procedure for US data, which was 2% (Chen & Petkova, 2012). This degree of ability of the model to explain excess market return one period ahead, according to Chen and Petkova (2012), "is comparable to other studies that analyze the predictability of monthly market return".

It is interesting to note that AV exhibits, according to the results reported in Table 2, a negative relationship with market excess in the following period, and a positive one with market variance in the following period. Campbell (1993) presents a description of how a shock in a variable that represents a reduction in expected market return indicates a worsening in conditions for investors. Chen (2003) extends this result and demonstrates that a worsening of investment conditions also depends on an increase in market variance. As a positive shock in AV indicates a reduction in excess expected market return and an increase in expected variance, this variable indicates a deterioration of future investment conditions, both in terms of expected return as well as risk. A variable with these characteristics should command a risk premium. Assets that respond well when positive shocks in AV occur serve as a hedge in poor market conditions and, therefore, should have a lower expected return. According to Chen and Petkova (2012), assets that respond well to positive shocks in AV are assets with high investments in research and development. This type of investment, due to its innovative character, offers alternatives for periods of crisis, meaning this asset/portfolio serves as a hedge. Consequently, its expected return will be lower (negative risk premium).

Table 2 - Regressions in temporal series

	1	**2**	**3**	**4**	**5**	**6**
Constant	-0.0015*	-0.0045*	-0.0109*	-0.065*	-0.0057	-0.0156
	(-2.73)	(-4.05)	(-2.0817)	(-3.1863)	(-1.4387)	(-0.8317)
AV_t x AC_t	1.1765*					
	(16.2923)					
AV_t		0.8219*		0.7951*		
		(9.1061)		(7.0594)		
AC_t			0.0394*	0.0056		
			(2.7036)	(0.8554)		
AV_{t-1}					0.2753*	-0.8530*
					(2.3124)	(-2.4590)
AC_{t-1}					0.0190	0.1010*
					(1.8643)	(2.2931)
R^2	0.896	0.814	0.211	0.818	0.200	0.100

Regressions in temporal series: contemporary (columns (1) and (4)) and predictive (5) for market variance, Vm, and predictive (6) for excess market return, Rm. The explanatory variables are AV x AC, AV, and AC. t statistics from Newey-West with four lags are in brackets. The asterisk indicates significance of 5% or less.
Source: Developed by the authors.

Table 3 below summarizes the average and the standard deviation of the factors used to capture the sensitivity (loads) of portfolios sorted by company size and by idiosyncratic volatility. Subsequently, these loads will be used to explain the price of risk of each of these factors in relation to the same portfolios. Rm is excess market return, while HML and SMB are the traditional factors from the Fama and French (1996) model. The variations in the market variance components, ΔAV and ΔAC, were calculated as described in item 3.5.

Table 3 - Average and correlation of factors

	Average	Stand. Dev.	HML	SMB	ΔAV	ΔAC
Rm	0.0172	0.0636	0.0187	-0.0207	0.0657	-0.1055
HML	0.0031	0.0470		0.1276	0.2501	0.0049
SMB	0.0003	0.0437			-0.5072	-0.1317
ΔAV	3.1911	1				0.0000
ΔAC	0.7424	1				

Presents the sample average, the standard deviation and the correlation for the Fama and French factors, Rm, HML, and SMB, and the innovations in average variance, AV, and in the average correlation, AC. The innovations in AV and AC derive from the orthogonalized and normalized VAR described in item 3.5.
Source: Developed by the authors.

Table 4 presents the coefficients that indicate the sensitivity of each portfolio to the factors from the Fama and French (1996) model, increasing by the variation in average variance, ΔAV, and by the variation in average correlation, ΔAC. The values indicate that the model adjusts well, and that the factors that are clearly more important for the Brazilian market are: excess market return, Rm; and the company size factor, SMB.

With regards to the average variance factor, assets that perform well in periods of market deterioration should have a positive load in relation to a variation in AV, since this variable predicts an increase in volatility and a reduction in average market return (cf. Table 2). Inversely, assets with weak performance in periods of crisis should exhibit a negative load in relation to a variation in AV. If the Brazilian data reproduced the US results, loads with changed signs would be expected for portfolios with high and low idiosyncratic volatility, however, in contrast to the US market, this is not verified, according to the results find in this study. Chen and Petkova (2012) verify that, in the American economy, companies with high (low) IV exhibit a high (low) level of investment in research and development (R&D), which is considered as an indication of the presence of real options. According to the literature, the value of a real option rises with an increase in volatility of the underlying asset. This fact would explain, for the US market, the characteristic of companies that have high (low) IV performing better (worse) in periods of crisis, and consequently, having positive (negative) sensitivity to the AV variation factor. In Brazil, almost all of the portfolios exhibited a negative load both in relation to AV variation as well as AC variation, with almost all also being statistically insignificant, despite these factors improving the explanatory power of the model. The conclusion which is drawn is that, for the Brazilian market, idiosyncratic volatility does not derive from sources that mitigate effects of a worsening in conditions for investors, i.e., it does not derive from R&D.

Another important point is that the ΔAV loads should rise from the portfolios with lower IV to those with higher IV, as predicted in Equation 6, which is also not verified in the Brazilian case.

The explanation for this difference between Brazil and the United States may be in the culture of investment in R&D. Trademarks and patents registration in Brazil is substantially lower than in the United States. The number of patents registered by each country was taken as a proxy for the volume of investment in R&D, according to data from the *PCT Yearly Review* (2014), from the World Intellectual Property Organization (WIPO), an agency of the UN. The total patent requests filed by the United States in 2013 were approximately 57,000, with 85% being from private companies. In the same period, Brazil filed approximately 620 patent requests, with only 50% of this total being of private origin. Chen and Petkova (2012) cite this factor as one of the main ones for mitigating negative effects in periods of crisis. According to the authors, companies with a high level of R&D would serve as a hedge in periods of market deterioration, which would lead investors to accept paying a premium for them. This effect would cause a differentiation in the price of risk of these assets; their sensitive loads to the factors that predict a worsening in the market would be positive, that is, they would perform better than other companies in poor scenarios, leading to a negative price of risk, that is, a reduction in their expected return, since they would be seen as lower risk companies.

None of the above effects were identified in Brazil. The assets' indicative sensitivity loads to a worsening in the market are practically all statistically null, as can be observed in Table 4. This would indicate that there would not exist assets with better or worse average performance in poor periods, in relation specifically to this risk factor. The average price of risk identified for this factor was significant – as can be seen in Table 5 – and positive, that is, the opposite of what was found for US data.

These results seem coherent with the theory defended by Chen and Petkova (2012). According to these authors, in the United States, companies with high (low) IV exhibit a high (low) level of investment in R&D; such investment is perceived by market participants as a hedge for periods of high volatility and, consequently, a lower expected return is demanded – negative premium. In Brazil, the level of this type of investment is very low, and in the absence of this risk mitigating factor, the risk premium is positive, that is, financial agents perceive IV as a real risk, due to it not being composed of factors that allow better performance in periods of higher volatility. Thus, the greater exposure of this risk factor prices a higher expected return and vice versa – positive premium. The results obtained in this study record that the hedge effect observed in the United States – generated, according to Chen and Petkova (2012), primarily by investments in R&D – is not reproduced in Brazil.

Table 4 - Coefficients of the expanded Fama and French model

α0						**βRm**					
	High IV	2	3	4	Low IV		High IV	2	3	4	Low IV
Large	-0.0034	0.0145	0.0300	0.0116	-0.0041	Large	0.9482*	0.9766*	0.9899*	0.9628*	1.0096*
2	0.0200	0.0246	0.0407	0.0353	0.0305	2	0.9833*	0.9787*	0.9883*	0.9798*	0.9848*
3	-0.0065	0.0235	0.0148	0.0367	0.0089	3	0.9690*	1.0097*	1.0325*	1.0115*	0.9941*
4	-0.0200	0.0040	0.0057	0.0210	0.0145	4	0.9571*	0.9652*	0.9839*	0.9881*	0.9861*
Small	-0.0023	-0.0047	0.0341	0.0353	0.0086	Small	0.8241*	0.9739*	0.9884*	1.0008*	1.0131*

βSMB						**βHML**					
	High IV	2	3	4	Low IV		High IV	2	3	4	Low IV
				-							-
Large	-0.1577	0.0654	-0.1018	0.2877*	-0.0559	Large	0.4609*	0.1970	0.2928	0.1817	0.1997*
2	0.8853*	0.3421*	0.1579	0.0061	0.0038	2	0.4042*	0.3637	0.2664	0.4226*	0.2266
3	1.128*	0.9124*	0.7631*	0.7455*	0.5004*	3	-0.0957	0.1854	-0.1355	0.0065	0.1247
4	1.5281*	0.9108*	0.7712*	0.5584*	0.5653*	4	-0.0696	0.1095	0.1334	0.0314	-0.0172
Small	2.3554*	1.1761*	0.8774*	0.6702*	0.6342*	Small	0.9685	0.1920	0.0655	0.4381*	-0.0643

βΔAV						**βΔAC**					
	High IV	2	3	4	Low IV		High IV	2	3	4	Low IV
Large	-0.0165	-0.0104	-0.0106	-0.015*	0.0030	Large	-0.0003	-0.0031	-0.0099	-0.0068	0.0018
2	-0.0086	-0.0119	-0.018*	-0.019*	-0.0170	2	-0.0050	-0.0048	0.0061	-0.0048	-0.0040
3	-0.0056	-0.0047	0.0038	-0.0070	-0.0060	3	0.0085	-0.0086	0.0033	-0.0127	-0.0014
4	0.0055	-0.0077	-0.0057	-0.0110	-0.013*	4	-0.0154	-0.0041	-0.0060	-0.012*	0.0028
Small	-0.0211	0.0044	-0.0158	-0.0134	-0.0025	Small	-0.0227	-0.0202	0.0028	0.0015	-0.0016

This table presents the constant and the regression loads of 25 the portfolios sorted by size and idiosyncratic volatility (IV). The betas are calculated in the full sample. The independent variables are the Fama and French factors with portfolios rebalanced monthly, plus the ΔAV and the ΔAC. The model is described in 3.5. The asterisk indicates significance of 5% or greater, based on t statistic from Newey-West with four lags. The sample covers the period starting in January 2003 and finishing in July 2014.
Source: Developed by the author.

Column 1 of Table 5 presents the prices of risk for the standard model. γ_0 is statistically significant and represents the pricing error in the model. Although it continues to be different to zero in the other regressions, the components of the expanded model are not tradable portfolios, so nothing can be said about their significance. When the factors regarding the market volatility components are inserted into the model, their explanatory power increases. $\gamma_{\Delta AV}$ exhibits significance in all the experiments, indicating that the ΔAV component is priced in excess return on the portfolios. Another interesting point is the relationship between ΔAV and ΔVm. As these two factors are orthogonal by construction, their ranges can be interpreted as the additional contribution of one in the presence of the other. In this case, ΔAV captures all of the relevance, confirming the indication presented in Table 2 that the ambiguous effect of the correlation component – predicting both an increase in excess market return as well as an increase in aggregate variance – could hinder the performance of ΔVm as a factor explaining excess return on assets. By separating Vm into AV and AC, we can exclude the contradictory effect of the AC factor and isolate the explanatory power of average variance.

Table 5 - Regressions in temporal series

	1	2	3	4	5	6
γ_0	-0.5889	-0.5934	-0.5800	-0.5888	-0.6090	-0.5896
	(-6.2895)	(-6.2255)	(-5.9436)	(-6.4878)	(-6.5254)	(-6.0769)
γ_{Rm}	-0.3861	-0.3976	-0.3920	-0.3863	-0.3627	-0.3821
	(-4.3756)	(-4.2086)	(-4.1925)	(-4.5462)	(-4.2123)	(-4.1766)
γ_{HML}	0.0076	0.0092	0.0172	0.0076	0.0173	0.0178
	(0.8847)	(1.5544)	(1.9211)	(0.8445)	(1.7730)	(1.8601)
γ_{SMB}	-0.0099	0.0143	0.0093	0.0099	0.0100	0.0096
	(2.1967)	(2.0275)	(2.0580)	(2.0346)	(2.0838)	(1.9248)
$\gamma_{\Delta AV}$		0.8430	0.9482		0.9148	0.9546
		(2.9175)	(3.2703)		(3.3694)	(3.3611)
$\gamma_{\Delta AC}$			0.1710			0.1324
			(0.6486)			(0.3941)
$\gamma_{\Delta Vm}$				0.1131	0.0678	0.0285
				(0.7625)	(0.6854)	(0.2284)
R^2	0.21	0.24	0.26	0.25	0.25	0.27

Fama and MacBeth regressions using excess returns on the 25 portfolios, sorted by size and idiosyncratic volatility. The betas are the independent regression variables and were calculated for all of the sample. Rm, HTL, and SMB refer to the Fama and French factors, calculated with rebalanced portfolios month to month. ΔAV and ΔAC are the innovations in average variance and in average correlation, calculated as described in 3.5. ΔVm refers to the innovations in market variance and was calculated in a similar way to ΔAV and ΔAC. t statistics, in brackets, adjusted for error in the variables, as in Shanken (1992).
Source: Developed by the authors.

2.4.2
Complementary Analyses

In this paper, the Fama & French (1996) model with three factors is used, taking into consideration the intention of comparing the results found for the Brazilian data with those found for the US data, detailed by Chen and Petkova (2012). Lately, the literature has recorded that five factor models have been shown to be better specified to describe returns (see Amihud, 2014). As we did not want to lose comparability, but, at the same time, sought to test robustness and present results in line with the more modern models, a brief reproduction of this paper's main result was added, i.e., the risk premium for the average volatility ($\gamma_{\Delta AV}$) and average correlation ($\gamma_{\Delta AC}$) components for portfolios ordered by idiosyncratic volatility, estimated based on the five factor model. Table 6 below reproduces the results presented in Table 5, and includes the two new factors, WML and IML, presented respectively by Carhart (1997) and by Amihud (2014), in the analysis. These new factors were kindly supplied by the Nefin – FEA/USP team (n.d.). The results are robust and similar to those presented for the three factor model. The $\gamma_{\Delta AV}$ exhibited is 1.12 – 0.94 in the three factor model – both statistically significant and positive. The $\gamma_{\Delta AC}$ is close to zero and not significant, as in the previous model.

Table 6 - Regressions in temporal series – Five factor model

	1	2	3	4	5	6
γ_0	-0.6045	-0.6002	-0.6652	-0.6579	-0.5700	-0.6334
	(-6.2594)	(-6.2717)	(-6.7164)	(-6.7826)	(-5.3483)	(-5.9926)
γ_{Rm}	-0.3684	-0.3727	-0.3064	-0.3135	-0.4029	-0.3380
	(-4.0957)	(-4.1767)	(-3.4958)	(-3.6498)	(-3.9234)	(-3.4814)
γ_{HML}	0.0087	0.0083	0.0060	0.0051	0.0077	0.0048
	(1.1164)	(1.0220)	(0.7859)	(0.6546)	(0.9394)	(0.6109)
γ_{SMB}	0.0096	0.0092	0.0080	0.0071	0.0085	0.0067
	(2.1238)	(1.9516)	(1.7490)	(1.5313)	(1.8024)	(1.4319)
γ_{IML}	0.0310	0.0316	0.0266	0.0271	0.0303	0.0264
	(1.1473)	(1.1778)	(0.9949)	(1.0103)	(1.1357)	(0.9868)
γ_{WML}	-0.4871	-0.4900	-0.4459	-0.4497	-0.5027	-0.4608
	(-3.3231)	(-3.3573)	(-2.9869)	(-3.0331)	(-3.3801)	(-3.0493)
$\gamma_{\Delta AV}$			1.1228	1.1527		1.2344
			(3.8169)	(4.0196)		(3.8030)
$\gamma_{\Delta AC}$			-0.0363		0.1226	0.0506
			(-0.1419)		(0.4261)	(0.1779)
$\gamma_{\Delta Vm}$		0.1357		0.0825	0.1121	0.0681
		(0.9947)		(0.6490)	(0.8683)	(0.5543)
R^2	0.30	0.32	0.34	0.32	0.33	0.33

Fama and MacBeth regressions using excess returns on the 25 portfolios, sorted by size and idiosyncratic volatility. The betas are the independent variables in the regression and were calculated for the whole sample. Rm, HML, SMB, and WML refer to the Fama and French factors. IML refers to the liquidity factor from Amihud (2014). ΔAV and ΔAC are the innovations in average variance and in average correlation, calculated as described in 3.5. ΔVm refers to the innovations in market variance and was calculated in a similar way to ΔAV and ΔAC. t statistics, in brackets, adjusted for error in the variables, as in Shanken (1992).
Source: Developed by the authors.

As well as the analysis presented above, robustness tests were also carried out related to the quintiles and to the periods used in constructing the portfolios by size and IV. The test portfolios in the paper, as described previously, are composed of assets sorted by size and divided into five groups – 5 quintiles – and, subsequently, sorted by IV and divided again into five groups – another 5 quintiles – forming 25 studied portfolios. The nomenclature 5x5 will be adopted to refer to this composition. Thus, portfolios using 3x5 divisions would have three groups of shares sorted by size, followed by five groups of shares sorted by IV, forming 15 test portfolios in total. In order to evaluate robustness, different construction alternatives were elaborated for the test portfolios, such as 3x5, 3x6, 4x5, 4x6, and 5x6. Separate evaluations were also elaborated in the first and in the second half of the sample. As a whole, the values found were shown to be robust. Altering the composition of the portfolios, the difference (spread) of average IV between the portfolios was also altered, thus modifying the sensitivity of the factors regarding volatility and correlation and, consequently, influencing the magnitude of premium attached to these factors. However, the effect of the factors studied over the return on the portfolios was not altered, i.e., the average volatility component ($\gamma_{\Delta AV}$) premium is positive in all of the tests, while the average correlation ($\gamma_{\Delta AC}$) component is statistically insignificant. These results align with the paper's main results. In Table 7 some tests considered representative were reported; the rest, not reported, indicate quite similar values to those presented.

Table 7 - Regressions in temporal series – Robustness tests

	4x4	4x6	4x4 – 1st part	4x4 – 2nd part	4x6 – 1st part	4x6 – 2nd part
γ_0	-0.8396	-0.8245	-1.1015	-0.7090	-0.9898	-0.7251
	(-8.382)	(-9.6806)	(-7.4717)	(-7.5468)	(-7.5335)	(-7.7075)
γ_{Rm}	-0.1322	-0.1484	-0.0918	-0.0520	-0.2053	-0.0360
	(-1.8181)	(-2.7636)	(-1.3394)	(-1.2588)	(-3.1626)	(-0.9815)
γ_{HML}	0.0140	0.0099	0.0046	0.0022	-0.0060	0.0011
	(1.6559)	(1.1231)	(0.3865)	(0.2749)	(-0.5323)	(0.1511)
γ_{SMB}	0.0041	0.0054	0.0130	0.0090	0.0144	0.0079
	(0.9247)	(1.2481)	(1.6671)	(1.7612)	(1.8569)	(1.5496)
$\gamma_{\Delta AV}$	0.5924	0.6278	0.4147	0.2562	0.7330	0.2000
	(2.2912)	(3.2656)	(1.8337)	(1.6730)	(3.5177)	(1.4022)
$\gamma_{\Delta AC}$	-0.1920	-0.1858	-0.3862	-0.0685	-0.3716	-0.3142
	(-0.6876)	(-0.7725)	(-1.2648)	(-0.2949)	(-1.3489)	(-1.1744)
R^2	0.31	0.26	0.36	0.28	0.30	0.26

Fama and MacBeth regressions using excess returns on different portfolios sorted by size and idiosyncratic volatility. The betas are the independent variables in the regression and were calculated for the whole sample in columns 4x4 and 4x6. The rest, in accordance with that indicated in the table – 1st and 2nd parts of the sample. Rm, HML, and SMB refer to the traditional Fama and French factors. ΔAV and ΔAC are the innovations in average variance

and average correlation, calculated as described in 3.5. t statistics, in brackets, adjusted for error in the variables, as in Shanken (1992).
Source: Developed by the authors.

2.5 Final Discussions

In this study we used the fact that an asset's idiosyncratic volatility – defined as the standard deviation of residuals in the Fama and French (1996) model – is directly affected by the absence of an explanatory factor in the model, in direct proportion to the sensitivity of the asset to the absent factor. Thus, idiosyncratic volatility can be seen as a proxy for a risk factor, in accordance with Chen and Petkova (2012).

Following, therefore, the intuition of Ang et al. (2006) that market aggregate volatility is priced, even though it exhibits contradictory behavior, and of Chen and Petkova (2012), who, in order to explain this contradiction, propose to break aggregate volatility up into average variance and average correlation, it was analyzed whether idiosyncratic volatility is priced in the Brazilian market.

It was identified that the average variance component predicts a reduction in excess market return and an increase in variance, thus being a sign of deterioration in investment conditions. Average correlation exhibits ambiguous behavior, predicting an increase in excess return and an increase in variance. These results are consistent with the international literature. Thus, the decomposition of volatility into the two components allows that average variance can better price the effects of a worsening or improvement in the investment environment, without the disturbance generated by the correlation component. These results are also identical to those found for US data, indicating that in Brazil, like in the United States, the average variance component should command a risk premium in relation to portfolios sorted by size and IV.

It occurs that, for US data, the risk premium commanded by average variance is significant and negative. The main explanation indicated by Chen and Petkova (2012) for a negative premium is the high level of investment in research and development by companies with a high level of IV. Portfolios composed of these companies would act as a hedge against deterioration of the environment and, thus, would have lower returns expectations. As the volume of investment of research and development recorded in Brazil is significantly reduced, if compared with that recorded in the United States, the expected result was that the Brazilian premium was positive. In fact, this occurs, and the risk premium commanded by exposure to average variance, according to the results found, is statistically significant and positive.

3
Size, Value, Profitability, and Investment: Evidence from Emerging Markets

3.1
Introduction

Since its first appearance in 1993, we have seen many works on Fama and French's (FF; 1993) model, but most relate to developed countries and/or to developed regions. For example, Chan, Hamao, and Lakonishok (1991) show a representative value premium in Japan's stock market. Fama and French (1998) study returns on market, size, and value portfolios for the US and Europe, Australia, and Far East countries and find a significant difference between high and low B/M stock returns in twelve of thirteen major markets. Again, Fama and French (2012) test size, value, and momentum for North America, Europe, Japan, and Asia-Pacific markets, finding a value premium that decreases with size, except for Japan, and a return momentum in every region. These applications provide many interesting insights about portfolio returns, but still fail to find a perfect statistical fit for the data. In 2015, Fama end French (2015) propose two more factors, a profitability factor, RMW (robust minus weak), and an investment factor, CMA (conservative minus aggressive) to overcome this fact. This new composition still rejects the null hypothesis of zero alphas in the linear regressions for excess returns, but capture the pattern of average stock returns better than the three-factor model does. They also find that the value factor became redundant in light of these two new factors due to its close relationship.

Following this line of work, far fewer studies use emerging market data. Many facts challenge existing studies and the models' efficiency in this kind of environment: low quality data, small markets, border hazards, political instability, and market fragility related to international speculative capital flows, among others. Bekaert and Harvey (1995) and Harvey (1995) were the first to look at emerging market issues. Fama and French (1998), Griffin et al. (2003), and Cakici et al. (2013) followed suit, among a few others. These studies mostly examined the original three-factor model and a four-factor model, adding a momentum factor (Carhart, 1997) to the original and studying its effects on emerging stock markets.

To the best of our knowledge, our study is the first to construct the RMW and the CMA ratio explanatory factors for emerging markets, as well as the cross-section for portfolios constructed on these indicators. Our study aligns with many others that focus on the US and developed markets (see, e.g., FF, 1993, 1998, 2015; Griffin 2002; Rouwenhorst, 1998), emerging markets, such as Cakici et al. (2013), and aggregate market returns (see, e.g., Bekaert and Harvey, 2002; Bekaert and Harvey, 2003).

Our main objective is to test a well-accepted financial asset pricing methodology for twelve main emerging markets divided into three emerging economic blocks. We seek to verify if our results are close to those found for developed markets. We deal with many problems that affect the results from such samples, including small sample sizes, low portfolio diversification, political instability, economic problems, and border hazard, among others. Despite these issues, we find results that, if not as strong and clear as those for developed markets, show some evidence that they follow the same trends found in US data and other developed countries. These are encouraging results and much better than we expected. This is important because it shows that emerging market players are demanding premiums for the same risk factors as those of developed markets. Thus, other kinds of influences the models do not address, like political issues, for example, are becoming less important, which leaves room for a more consistent and reliable economy. International players would like to see this; it provides an incentive that attracts international investments, which is a very important means to help many of these countries overcome poverty. These economic blocks are home to almost half of the world’s population, and we would like to see a solid economic base for investments to make it easier to improve all of those people's lives. When we find that the portfolio-pricing patterns follow the same trends as in developed economies, we see that players in emerging economies are pricing assets rationally, that is, the variables related to average returns are proxies for sensitivity to common risk factors in returns (see Fama and French, 1993). Our results show that a distance remains between emerging and developed markets in terms of the factors of risk driving players’ expectations. However, most of the observed results are similar to those for developed countries, which implies that the perception of risk factors driving portfolio excess returns in emerging markets is getting closer to that of developed countries, that is, emerging financial markets are working better. In the following sections, we discuss the results of the Fama and French three-, four- and five-factor models for emerging markets and the differences and similarities of these results in respect to those in the literature for developed markets. In this way, we believe that we further our understanding of the subject and provide new evidence for asset pricing in international emerging markets.

3.2
Data

In this study, we use a list of stock prices, market caps, equities, and EBITDAs from twelve countries: Brazil, Chile, Mexico, Argentina, India, China, Thailand, Malaysia, Turkey, Poland, Romania, and Russia, taken from the Bloomberg database divided into Latin America (LA), Asia, and Eastern Europe (EE) regional blocks. The sample runs from July 2007 to February 2017. We use 2007 and 2008 to obtain the first reference values for the ratios to build up the portfolios and Fama and French (2015) factors. We calculate the average monthly returns for the portfolios and factors for the 98 months, from January 2009 to February 2017, in US dollars. We use the one-month constant maturity T-Bill from the Federal Reserve Economic Data (FRED) as the risk-free rate. Our database includes historical data for firms that disappear, but it does not include historical data for new firms, so there is no survivor bias nor any backfilling problems.

We use a sample from 2009 to 2017 for two reasons. First, the dataset for some countries (all from LA and some from EE) available before 2008 is very small, that is, there are very few liquid stocks traded on their Stock Exchanges. Even after 2008, we some countries were still in this situation, as Table 1 shows. It is common in emerging markets that many stocks available for trading are not liquid. We frequently find stocks that have not been traded in years. These stocks' prices frequently do not represent the real value of the companies. Thus, use only stocks that traded for at least 80% of business days. For some of the countries of interest, this number is very low before 2008. The other reason is that we would like to understand the effects of the factor after the global financial crisis of 2008, as we believe this established a new relationship among international markets.

Tables 1.a, 1.b, and 1.c describe the number of firms and market cap for each country at the beginning, middle, and end of the sample. We show only the data used to calculate the portfolio returns and factors, that is, liquid stocks as defined above, and with a positive book value for each year. We notice here that the difference between all stocks in each country's Stock Exchange and the liquid and book-positive stocks may be relevant. In LA, for example, we have more than 1,000 stocks each year for the whole sample, but only about 419 liquid stocks for each year on average. The same holds for the other two blocks. In Asia, more than 5,000 stocks from the original data set, but only about 2,851 liquid stocks exist per year on average. For EE, we find more than 2,500 stocks and only 760 liquid stocks per year on average. Thus, we can only conduct a complete study for the economic blocks; we do not have enough stocks for some countries to assemble diversified portfolios. Table 1.a shows that it would be impossible to assemble 25 portfolios from Mexico and Argentina with more than two stocks in some portfolios. For Russia and Romania, we have only two or three stocks in each portfolio. Consequently, we assemble the factors for each country, but remain conscious that for some, the portfolios will have very poor diversification. On the other hand, this reflects the reality in these countries and we show the factors calculated as best as we possibly can.

Table 1.a

Firms' characteristics of Latin American emerging markets. The table provides the number of firms in our data sample and the total market capitalization for each country. The market capitalization is from the last day of each year. We compute these values only with firms really used for computations: firms with liquidity (traded at least in 80% of trading days, each year) and positive book value.

	Brazil	Mexico	Chile	Argentina	Lat. Am.
Panel A: Number of Firms in Country					
2009	159	44	144	34	381
2012	186	51	160	26	423
2016	173	71	155	53	452
Ave	173	55	153	38	419
Panel B: Average Size (Market Capitalization, $ B)					
2009	2.90	4.60	0.73	0.85	2.09
2012	6.07	6.70	1.50	1.40	4.13
2016	2.54	4.60	0.98	0.92	2.14
Ave	3.84	5.30	1.07	1.06	2.79

Table 1.b

Firms' characteristics of Eastern European emerging markets. The table provides the number of firms in our data sample and the total market capitalization for each country. The market capitalization is from the last day of each year. We compute these values only with firms really used for computations: firms with liquidity (traded at least in 80% of trading days, each year) and positive book value.

	Turkey	Poland	Russia	Romania	Est. Euro
Panel A: Number of Firms in Country					
2009	248	273	78	69	668
2012	275	329	113	34	751
2016	331	382	100	48	861
Ave	285	328	97	50	760
Panel B: Average Size (Market Capitalization, $ B)					
2009	0.45	0.27	3.24	0.08	0.66
2012	0.67	0.37	5.53	0.24	1.25
2016	0.53	0.33	3.26	0.55	0.76
Ave	0.55	0.32	4.01	0.29	0.88

Table 1.c

Firms' characteristics of Asian emerging markets. The table provides the number of firms in our data sample and the total market capitalization for each country. The market capitalization is from the last day of each year. We compute these values only with firms really used for computations: firms with liquidity (traded at least in 80% of trading days, each year) and positive book value.

	India	China	Malaysia	Thailand	Asia
Panel A: Number of Firms in Country					
2009	1,129	511	346	308	2,294
2012	1,355	693	494	424	2,966
2016	1,372	772	607	542	3,293
Ave	1,285	659	482	425	2,851
Panel B: Average Size (Market Capitalization, $ B)					
2009	0.46	0.43	0.43	0.29	0.43
2012	0.72	0.33	0.74	0.59	0.61
2016	1.04	2.19	0.59	0.59	1.15
Ave	0.74	0.98	0.59	0.49	0.73

3.3 Three- and Five-Factor Models

For the well-known three-factor model of Fama and French (1993), we propose that the expected return on a portfolio in excess of the risk-free rate is explained by:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + s_i SMB_t + h_i HML_t + \varepsilon_{it} \quad (1)$$

Where, R_{Mt}, SMB_t, and HML_t are the excess returns on a market portfolio, the size factor (small minus high market cap), the value factor (high minus low B/M ratio), and α_i is a type of bad model specification indicator.

Many studies show that the average returns are correlated with the B/M ratio. Firms with high B/M values tend to have persistent low earnings and positive slopes on high minus low (HML) factors. Firms with low B/M values tend to present high earnings and negative slopes on HML (FF, 1996). Fama and French (2015) use the dividend discount model to show that profitability and investment add to the description of average returns that B/M provides. They show, as in Miller and Modigliani (1961) that total market value of a firm at time *t* is:

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau} \quad (2)$$

where, $Y_{t+\tau}$, is the total equity earnings for period $t+\tau$ and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity. They divide this value by time *t* book equity,

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \quad (3)$$

From this equation, they make three statements:

(1) Everything fixed in (3) except M_t and *r* tells us that a lower M/B implies a higher expected return;

(2) Everything fixed in (3) except Y_t and *r* tells us that higher expected earnings implies a higher expected return;

(3) Everything fixed in (3) except dB_t and *r* tells us that higher expected growth in book equity—investment—implies a lower expected return.

Those statements led the authors to examine a model that adds investment and profitability factors to their prior model. Therefore, they add two more factors to the three-factor model:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \quad (4)$$

where RMW is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA is the difference between the returns on diversified portfolios of stocks of low and high investment firms—conservative and aggressive. The remaining factors are the same as in equation (1).

Fama and French (2015) examine the performance of the five-factor model using US stock markets data (FF, 2015). We use the same motivation to investigate how the model works with emerging markets stock data and to test its integration with US and global markets.

3.4
Time Series Regression Inputs

3.4.1
Explanatory factors

We can see in equations (1) and (4) that the three- and five-factor models describe the excess returns on the assets. On the RHS of the equations, the first factor is the excess return on the market portfolio, the market portfolio returns minus the risk-free rate, called the market factor (R_{Mt}). The second factor is the spread between the returns on portfolios with small and big companies considering the market cap value, or the small minus big factor (SMB_t). The next factor is the spread between the returns on portfolios with a high and low B/M ratio, or the high minus low factor (HML_t). Following the sequence, we can see the robust minus weak factor (RMW_t), which is the difference between the returns on companies with high operational profitability (OP) minus companies with low OP. The last factor is the conservative minus aggressive factor (CMA_t). This factor is the difference between returns on portfolios of low and high investment firms.

We calculate the factors in two ways, following the methods described in Fama and French (1993, 2015). We always obtain the most recent data to guarantee that the data is available for the market before constructing the portfolios.

In the first method, we build six portfolios of stocks sorted on market cap (size) and B/M, size and OP, and size and investment (Inv). For each year *t*, we use the market cap from December of *t*-1 to divide the sample in two. Stocks above the median are called the big portfolio (B) and those stocks below the median are called the small portfolio (S). We then sort each of these portfolios by B/M ratio using the book value in July of *t*-1 and the market value in December of *t*-1. We then divide both portfolios in three (2x3). The companies situated among the lowest 30% B/M enter the low group (L); those among the middle 40% enter the medium group (M); and finally, those among the highest 30% enter the high group (H). We then get six groups ordered by size and B/M—SL, SM, SH, BL, BM, and BH. We repeat this procedure for OP instead of B/M, dividing EBITDA in July of *t*-1 by the book value in July of *t*-1. We then repeat this for investment (Inv), which is the difference between the book value in July of *t*-1 and the book value in July of *t*-2 divided by the book value in July of *t*-2. We get three sets of six portfolios. Then, we calculate the value-weighted returns of these portfolios from January to December of year *t*, when we redefine the portfolios using our methodology. We generate three SMB factors based on the average returns of the small portfolios (SL, SM, SH) and the average returns of portfolios of big companies (BL, BM, BH) and form SMB-BM, SMB-OP, and SMB-Inv portfolios. Finally, we obtain the returns on the SMB factor as the average return of these three portfolios.

In the second method, we follow the same reasoning, but split the stocks into two portfolios at a time sorted by each of the ratios cited above. Therefore, we sort the sample at year *t*-1 by market cap and divide it into two groups: big (B) and small (S). We divide these two groups into four, sorting the two portfolios by B/M and then dividing each of these into two other portfolios. Then, we obtain big high (BH), big low (BL), small high (SH), and small low (SL) portfolios, which we sort by OP and divide them into eight portfolios according to the high profitability, which we call robust (R), and low profitability, termed weak (W). Finally, we sort each of the eight portfolios by Inv and the sample into sixteen new portfolios according to its investment policy: we call weak investment firms conservative (C) and strong investment firms aggressive (A). We then obtain BHRC, BHRA, BHWC, BHWA, BLRC, BLRA, BLWC, BLWA, SHRC, SHRA, SHWC, SHWA, SLRC, SLRA, SLWC, and SLWA (2x2x2x2) portfolios. From these portfolios, we find the factors' returns using SMB as the average return of the S portfolios—all portfolios with an S in its name, minus the average returns of the B portfolios, that is, all portfolios with a B in its name. The HML is the average returns of the H portfolios minus the average return of the L portfolios, and so on.

Both methods are identical to those described in Fama and French (2015). Following this procedure, we find two sets of five factors. The first we call the 2x3 factors and the second one the 2x2x2x2 factors. We show their summary statistics by country in Table 2 and for the three economic blocks in Table 3.

Unfortunately, India is the only country that has enough liquid stocks, that is, stocks traded in at least 80% of trading days and with a positive book value for each year, to build truly diversified portfolios in a 5x5 sort. We can have more than

45 stocks in each portfolio every month of the period. For the other Asian countries, we have only 20-25 stocks on average. For EE, we have 10-15 stocks in the Turkey and Poland portfolios, and 2-4 stocks in Russia and Romania on average. As we said above, this is much less diversification than we would like. However, this is their reality and we calculate the factors as well as possible to show this reality.

The average returns range from -1.24% to 1.36% and -1.10% to 1.28% for the 2x3 SMB and 2x2x2x2 portfolios, respectively. The average SMB returns are similar for both versions of the factor in the country-wise analysis. In fact, SMB is the factor with the biggest correlation between the two versions for the economic region calculation, as we can see in Table 3, ranging from 0.94 to 0.99. The difference between the SMB of the 2x3 and 2x2x2x2 portfolios is that the first is calculated as described in Fama and French (1993) and the second as in Fama and French (2015).

Table 2

Average, standard deviations and t-Statistics monthly factors percent returns from january, 2009 to february, 2017, for 98 months. All the returns are in US dollars. We calculate all five factors with two different methodologies described in Fama e French, 2015. For the 2x3 factores we take the stocks ordered by market capitalization and divided them into two portfolios - Big and Small. SMB is the difference between the returns on difersified portfolios of small stocks and big stocks. Then, we sort each portfolio - Big and Small - by book-to-market ratio, proftability and investment level and divided them into three new portfolios - High, Neutral and Low. HML is the difference between the returns on diversified portfolios of high and low B/M stocks. RMW is the difference between the returns on diversified portfolios of sotcks with robust and weak proftability. CMA is the difference between the returns on diversified portfolios of the stocks of low and high level of investment. For the 2x2x2x2 factors, we divided data into two portfolios at a time, first by Size - Big and Small; then, the previous two are sorted by B/M and divided into other two, and we get four portfolios. Then we divide each portfolio into two, sorting by Proftability and we get eight portfolios. Finally, we divide each portfolio into two, sorting by Investment and we get sixteen portfolios. SMB is the average returns on the eight portfolios of small stocks minus the average returns of the eight portfolios of big stocks. HML is the average returns on the eight portfolios of high B/M stocks minus the average returns of the eight portfolios of low B/M stocks. RMW is the average returns on the eight portfolios of robust proftability stocks minus the average returns of the eight portfolios of weak proftability stocks. CMA is the average returns on the eight portfolios of low investment stocks minus the average returns of the eight portfolios of high investment stocks. Rm - Rf is the difference bwtween the value-weight market returns and the one-month treasury bill rate.

	2 x 3 Factors					2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*	*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Brazil										
Mean	0.49	0.28	-0.30	-0.09	0.22	0.49	0.30	-0.31	0.06	-0.32
Std. Dev.	7.24	5.93	7.16	4.40	5.42	7.24	6.15	5.77	4.21	4.47
t-Statistic	0.67	0.46	-0.41	-0.21	0.41	0.67	0.49	-0.54	0.15	-0.72
Chile										
Mean	1.85	1.21	1.91	1.17	0.95	1.85	1.28	1.72	1.30	0.89
Std. Dev.	18.75	18.05	19.85	19.43	16.64	18.75	18.16	18.98	18.23	17.55
t-Statistic	0.98	0.66	0.95	0.60	0.57	0.98	0.70	0.90	0.70	0.50
Mexico										
Mean	0.62	0.03	-0.17	0.02	0.00	0.62	-0.18	-0.56	-0.68	-0.26
Std. Dev.	5.05	5.51	6.16	4.31	5.04	5.05	5.83	4.32	3.81	4.05
t-Statistic	1.22	0.06	-0.28	0.05	0.00	1.22	-0.30	-1.28	-1.77	-0.63
Argentina										
Mean	1.93	-0.51	-1.09	-1.91	-2.20	1.93	-0.87	-0.89	-1.56	-2.73
Std. Dev.	9.96	6.20	7.72	7.61	8.00	9.96	6.27	6.01	5.63	7.08
t-Statistic	1.92	-0.82	-1.40	-2.49	-2.72	1.92	-1.37	-1.46	-2.74	-3.82
China										
Mean	1.05	1.36	0.13	-0.07	0.12	1.05	1.23	0.18	-0.05	-0.03
Std. Dev.	5.97	4.48	3.93	1.80	1.75	5.97	3.67	2.76	1.40	1.42
t-Statistic	1.74	3.01	0.34	-0.39	0.66	1.74	3.32	0.65	-0.32	-0.24
India										
Mean	0.26	0.59	-0.65	-0.03	-0.41	0.26	0.55	-0.51	0.22	-0.45
Std. Dev.	5.92	5.78	6.34	2.64	2.73	5.92	5.60	4.51	2.55	2.74
t-Statistic	0.43	1.01	-1.02	-0.13	-1.49	0.43	0.98	-1.13	0.85	-1.61
Malaysia										
Mean	0.52	0.79	0.23	0.07	0.13	0.52	0.61	0.21	0.19	-0.30
Std. Dev.	4.01	4.37	4.35	2.66	2.62	4.01	4.09	3.63	2.11	2.98
t-Statistic	1.27	1.78	0.52	0.26	0.50	1.27	1.48	0.58	0.88	-0.99
Thailand										
Mean	1.42	0.66	-0.89	-0.02	-0.43	1.42	0.58	-0.92	0.42	-1.14
Std. Dev.	5.58	4.48	10.40	6.35	5.82	5.58	5.37	9.22	5.31	6.35
t-Statistic	2.52	1.46	-0.85	-0.03	-0.74	2.52	1.06	-0.99	0.79	-1.78
Russia										
Mean	0.73	-0.18	-0.87	0.18	-0.12	0.73	-0.33	-1.15	-0.52	-1.31
Std. Dev.	7.09	7.96	8.42	7.28	6.05	7.09	8.30	7.22	5.84	6.38
t-Statistic	1.02	-0.22	-1.02	0.25	-0.20	1.02	-0.40	-1.57	-0.88	-2.03
Turkey										
Mean	0.68	-1.24	-0.26	-2.72	1.02	0.68	-1.10	-0.78	-0.95	-1.25
Std. Dev.	8.59	9.87	6.25	21.89	18.08	8.59	7.02	6.79	6.90	6.60
t-Statistic	0.78	-1.24	-0.42	-1.23	0.56	0.78	-1.56	-1.13	-1.36	-1.88
Poland										
Mean	0.74	0.04	-0.06	-0.30	-0.66	0.74	0.07	0.12	0.07	-0.95
Std. Dev.	7.65	5.54	4.85	4.65	4.06	7.65	5.49	4.94	3.76	4.09
t-Statistic	0.96	0.08	-0.13	-0.65	-1.62	0.96	0.13	0.24	0.19	-2.31
Romania										
Mean	1.07	0.80	-1.53	-2.10	-0.73	1.07	0.75	0.22	-0.33	-1.04
Std. Dev.	9.45	8.09	12.04	9.87	9.84	9.45	9.15	8.79	10.58	8.56
t-Statistic	1.13	0.99	-1.26	-2.11	-0.74	1.13	0.81	0.25	-0.31	-1.20

For HML, RMW, and CMA we can see a smaller standard deviation for the 2x2x2x2 sort for most countries. The 2x3 sort uses the average returns of two portfolios with 30% of all stocks in each to calculate the high and low B/M portfolio returns. In the 2x2x2x2 sort, we create 16 portfolios, each with 6.25% of the stocks. Then, we take the difference in the average returns of the two portfolios formed by the average returns of eight portfolios each out of the 16 originals. The difference in these two methodologies influences the diversification of the portfolios we use to calculate the factors. For HML, ten out of the twelve countries have a smaller standard deviation in the 2x2x2x2 sort than in the 2x3 sort. For RMW eleven, and for CMA, six are smaller and two are equal.

While building factors for the economic blocks, we find more diversified portfolios as the number of stocks available for the blocks are significantly greater than for each individual country. The correlation between the 2x3 and 2x2x2x2 factors decreases as we add new control variables. The 2x3 factors control for size and one other variable (HML, RMW, or CMA), while the 2x2x2x2 factors control for all four variables. As we apply each control variable, the portfolios of the next filtration step became more distant from the 2x3 composition, especially from the third sorting onward.

The mean return from the SMB and HML from the 2x3 sort is similar to that from the 2x2x2x2 sort. For LA, the values are 0.72% and 0.63% for SMB and 0.26% and 0.11% for HML (Table 3). For Asia, the values are 0.80% and 0.84% for SMB and 0.19% and 0.22% for HML. For EE, the different sorts boost the SMB premium and a drop in the HML premium. The mean SMB returns rise from 0.75% to 1.17% for SMB and drop from 0.13% to -0.42% for HML. Additionally, the same table shows that the correlation between SMB and HML from the two sort systems is very strong for LA and Asia, and weaker for EE. The correlations between SMB and HML for LA are 0.99 and 0.95, respectively, showing that the joint controls provided by the 2x2x2x2 sort criteria have little effect on the factors. We see the same for Asia, where the correlations are 0.98 and 0.92, respectively. However, for the EE factors, we see a lower but still high SMB correlation, of 0.94, and a slightly weaker correlation for HML of 0.82.

On the other hand, the lower correlations of RMW and CMA among all three regions, from -0.42 to 0.88, indicate that the joint controls make some difference. In all three regions, the profitability premium increases with joint controls. They increase from 0.01% to 0.21% for LA; from -0.16% to 0.05% for Asia, and from -0.24% to 0.19% for EE. For the CMA factor, the premium increases for EE and the drop for LA and Asia stocks, in the order of 0.05% to 0.17%, 0.22% to -0.33%, and 0.13% to -0.16%, respectively.

Another interesting finding is the relatively high correlation among the 2x3 factors for all three blocks. For example, -0.72 for SMB and RMW and -0.55 for SMB and CMA for LA, or the correlation of 0.78 between SMB and RMW in the Asia factors and 0.65 for SMB and CMA for EE countries. Perhaps these factors do not capture risks such as political risk and exchange risk, making the factors somewhat indifferent for investors in these regions.

Table 3

This Table provides for the economic blocks, the same informtion described at Table 2, plus some correlations. Panel A shows basic statistics for the five factors calculated the same way presented at Table 2. Panel B shows the correlation between different versions of the same factors. Panel C shows the pairewise correlation among all five factors. Data period is from january, 2009 to february 2017. All returns are in US dollars.

Latin America

Panel A: Average, standard deviations and t-Statistics for monthly returns

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.71	0.72	0.26	0.01	0.22	Mean	0.71	0.63	0.11	0.21	-0.33
Std. Dev.	6.60	4.65	5.81	4.76	3.76	Std. Dev.	6.60	5.48	4.18	3.33	5.62
t-Statistic	1.06	1.52	0.45	0.02	0.57	t-Statistic	1.06	1.14	0.27	0.63	-0.57

Panel B: Correlation between different versios of the same factors

	SMB	HML	RMW	CMA
	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2
2 x 3	0.99	0.95	0.88	0.83

Panel C: Correlation between different factors

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Rm-Rf	1.00					rm-rf	1.00				
smb	0.18	1.00				smb	0.20	1.00			
hml	0.41	-0.45	1.00			hml	0.36	-0.53	1.00		
rmw	-0.35	-0.72	0.38	1.00		rmw	-0.22	-0.78	0.58	1.00	
cma	-0.09	-0.55	0.69	0.57	1.00	cma	-0.12	-0.78	0.73	0.81	1.00

Asia

Panel A: Average, standard deviations and t-Statistics for monthly returns

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Mean	1.15	0.80	0.19	-0.16	0.13	Mean	1.15	0.84	0.22	0.05	-0.16
Std. Dev.	5.72	5.11	3.10	3.22	1.56	Std. Dev.	5.72	5.75	2.16	5.02	1.52
t-Statistic	1.98	1.54	0.61	-0.49	0.82	t-Statistic	1.98	1.44	0.99	0.09	-1.07

Panel B: Correlation between different versios of the same factors

	SMB	HML	RMW	CMA
	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2
2 x 3	0.98	0.92	0.79	0.55

Panel C: Correlation between different factors

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Rm-Rf	1.00					rm-rf	1.00				
smb	-0.09	1.00				smb	-0.07	1.00			
hml	0.31	0.25	1.00			hml	0.32	0.46	1.00		
rmw	-0.18	0.78	0.22	1.00		rmw	0.08	0.85	0.49	1.00	
cma	0.05	0.52	0.07	0.41	1.00	cma	-0.08	0.46	0.50	0.49	1.00

Eastern Europe

Panel A: Average, standard deviations and t-Statistics for monthly returns

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.83	0.75	0.13	-0.24	0.05	Mean	0.83	1.17	-0.42	0.19	0.17
Std. Dev.	7.95	5.99	6.51	7.63	4.81	Std. Dev.	7.95	6.60	5.18	3.39	4.36
t-Statistic	1.0346	1.2430	0.1977	-0.3160	0.0995	t-Statistic	1.0346	1.7527	-0.8116	0.5666	0.3819

Panel B: Correlation between different versios of the same factors

	SMB	HML	RMW	CMA
	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2	2 x 2 x 2 x 2
2 x 3	0.94	0.82	0.73	-0.42

Panel C: Correlation between different factors

	2 x 3 Factors						2 x 2 x 2 x 2 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Rm-Rf	1.00					rm-rf	1.00				
smb	-0.04	1.00				smb	0.02	1.00			
hml	0.28	-0.23	1.00			hml	0.05	-0.16	1.00		
rmw	-0.08	0.38	0.30	1.00		rmw	-0.06	0.26	0.47	1.00	
cma	-0.04	0.65	-0.11	0.54	1.00	cma	0.29	-0.34	0.32	-0.24	1.00

Table 4 reports the basic statistics for the US and global factors from Kenneth French’s website. We cut the sample to adjust it for the period and calculate the same factors for emerging markets. For this small sample, few factors are statistically significant and the correlations among factors are much lower than we find for the emerging blocks. We use this data to test market integration. We regress the 75 portfolios formed by Size-B/M, Size-OP, and Size-Inv on these factors and compare the results to the regressions of the same portfolios on local factors.

Table 4

Summary statistics for monthly factors percent returns from US market and Global Developed. Data are obtained from Kenneth French's website. These data are used to test for emerging markets segmentation. Some basic statistics and correlations are presented here. The factors are calculated the same way described in the main text and at Table 2. Data goes from january 2009 to february 2017.

Panel A: Average, standard deviations and t-Statistics for monthly returns

USA	2 x 3 Factors					Global Developed	2 x 3 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Mean	1.27	0.16	-0.01	0.08	0.11	Mean	1.01	0.15	-0.04	0.24	-0.03
Std. Dev.	4.15	2.48	2.78	1.58	1.41	Std. Dev.	4.41	1.38	1.85	1.15	1.19
t-Statistic	3.03	0.63	-0.04	0.53	0.76	t-Statistic	2.27	1.07	-0.24	2.10	-0.22

Panel B: Correlation between different factors

USA	2 x 3 Factors					Global Developed	2 x 3 Factors				
	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*		*Rm-Rf*	*SMB*	*HML*	*RMW*	*CMA*
Rm-Rf	1.00					Rm-Rf	1.00				
smb	0.43	1.00				smb	-0.02	1.00			
hml	0.40	0.31	1.00			hml	0.34	0.05	1.00		
rmw	-0.38	-0.39	-0.27	1.00		rmw	-0.44	-0.28	-0.52	1.00	
cma	0.12	0.19	0.59	-0.01	1.00	cma	-0.15	-0.10	0.42	-0.13	1.00

3.4.2 Returns

On the LHS of (4), we use 25 portfolios sorted on size and B/M (Value), size and OP, and size and Inv. We take the whole sample and sort it by size according to market cap. We divide it into five quintiles and then sort each quintile into five by B/M. We now have 25 portfolios. The value-weighted excess returns of these portfolios are the dependent variables in our regressions. We repeat this procedure to get 25 portfolios sorted by size and OP, and size and Inv. By the end of this process, we obtain 75 portfolios to run our regression model.

Tables 5.a, 5.b, and 5.c report the average returns in excess of the 1-month T-bill for the 75 portfolios for all 3 economic blocks. We see a size effect, on average, in almost every country individually (untabulated results), and for the three regional blocks. The value, profitability, and investment effects do not show the same regularity as the size effect does. The patterns are not as clear as they are for developed markets, but we expected this result. The economic and political fragility of many countries combined with the small number of liquid stocks helps to explain these results. The excess returns on these ratios does not show the same pattern as in developed countries in most of the results, except for the size effect.

Table 5.a for the Size-B/M portfolios shows average excess returns of 2.22%, 1.68%, 1.56%, 1.57%, and 0.99% for Asia portfolios from the small to big stocks. Small companies have higher average returns than big companies do, as expected. We can see the same pattern in the excess returns in the EE and LA portfolios on average. On the other hand, we expected the same kind of average pattern for portfolios going from low to high B/M, but this is not so. The EE block shows the inverse expected pattern of the B/M effect, as we can see in Table 5.a. LA and Asia show similar returns from the low to high B/M portfolios, which is not consistent with findings for developed markets. We would expect to find value companies showing a bigger excess return than growth companies do. Although the patterns in the risk factors' average returns are not all the same for emerging and developed countries, this result does not imply that the models do not follow the same trend for both kinds of economic blocks. Individually, the size factor follows exactly the same return pattern in both emerging and developed countries, which does not appear with the value and profitability factors, and appears partially for the investment factor, as we illustrate below. However, the three- and five-factor models, as we discuss in Section 5, follow the same trend as in the US and other developed markets, in their abilities to explain the average returns on portfolios. Of course, it would be much better to find similar patterns for all risk factors, which would indicate that emerging market economies were even closer to developed economies than we thought in terms of their asset-pricing rationality. This is not the case for all factors, but is the case for some, which is a good result considering all of the social, political, and economic differences between the two country groups. After the 2008 crisis, world economic activity was very low, with very little or decreasing economic growth in most countries. This could have led investors to disregard the effect of performance rates and focus on the economic situation overall. In other words, the effect of the world economic situation on stock prices would have been much greater than the effect of B/M and OP ratios. This would have made portfolio returns indifferent to these financial indicators. Company size would remain relevant; a small company would have been affected more by the world situation than a big company would be. On the other hand, for investment rate, we see some slight effect for LA and EE. That is, even with the crisis and the

low level of world economic activity, companies that invested more were seen as less risky and presented a lower excess return than companies with low level of investment did (Table 5.c).

We can see for the Size-OP portfolios in Table 5.b for LA, that the average return for the OP ratio are 0.24%, 0.34%, 0.38%, 0.10%, and 0.11% from the low OP to high OP portfolios. They show a much more constant pattern than a growing one, as we would expect. All three economic blocks have a similar pattern. Asia portfolios present even more constant behavior, with returns running from 0.89% to 0.88%, almost without oscillation.

We can see the investment ratio effect in Table 5.c. We expect to see low-investment portfolios with average return higher than high-investment portfolios, as equation (3) demonstrates. Empirically, we note a weak pattern for the LA and EE blocks on average. We have a decreasing sequence of average returns on LA portfolios, going from 0.50% to 0.02%, except for the 0.46% return. The same thing appears in EE, except the last return, which has a value of 0.94%.

As we mention above, we did not expect to see all effects clearly in all economic blocks due to these countries' economic and political realities and the consequences of the 2008 crisis. After the crisis, the world macro-economic situation was not propitious for emerging markets, which possibly affected how market participants interpreted the effect of these variables on portfolio returns. Moreover, for some countries, political turbulence and internal economic problems could have affected the pattern of firms' profitability and investment, as well as the will to invest. However, despite these problems, we can find a reasonable contribution of the factors generated with the ratios on the fitting of the four tested models, which Tables 6.a and 6.b show.

Table 5.a

The table reports average monthly excess return on 25 portfolios formed on Size and Book-to-Market ratio. We present results for the three economic block: Latin America, Asia and Eastern Europe. All returns are in US dollars. Data range from january, 2009 to february, 2017. Stocks are sorted into five size portfolios, from small to big, based in their market capitalization. Then, each portfolio is sorted again, into five new portfolios, based on their book-to-market ratio (B/M), from low to high. These two sorts produce 25 value-weighted Size-B/M portfolios. The table shows the averages of monthly returns in excess of the one-month US Treasury bill rates.

	Mean						Standard Deviation				
	Low	2	3	4	High	Avg	Low	2	3	4	High
Latin America											
Small	1.35	0.97	1.50	2.37	1.23	1.49	14.46	12.50	9.56	10.29	10.19
2	0.88	0.65	1.05	0.80	0.89	0.85	14.74	8.00	12.99	9.41	7.65
3	1.33	1.08	1.13	1.21	1.14	1.18	11.39	6.77	8.26	7.68	8.70
4	0.78	0.68	0.91	1.28	0.75	0.88	7.63	8.81	6.31	7.75	8.07
Big	0.85	0.36	0.86	0.52	0.65	0.65	6.73	5.48	6.66	7.69	9.54
Avg	1.04	0.75	1.09	1.24	0.93						
Asia											
Small	2.36	2.61	2.13	2.18	1.84	2.22	6.38	6.60	6.64	7.63	9.69
2	1.53	1.69	1.68	1.73	1.76	1.68	6.24	6.19	6.11	6.62	7.59
3	1.71	1.82	1.40	1.33	1.54	1.56	5.85	5.68	6.02	6.58	6.89
4	1.54	1.71	1.61	1.61	1.38	1.57	6.77	7.34	7.18	7.03	6.70
Big	0.83	1.07	0.67	0.95	1.44	0.99	5.14	5.90	6.30	6.35	6.50
Avg	1.60	1.78	1.50	1.56	1.59						
Eastern Europe											
Small	1.24	1.48	2.08	1.48	2.07	1.67	9.22	7.59	8.56	8.45	8.44
2	2.18	1.39	0.96	1.35	1.00	1.38	15.46	6.90	7.67	8.49	8.14
3	1.57	0.97	1.33	1.32	0.88	1.21	10.91	6.89	7.59	8.48	9.00
4	1.32	1.08	1.18	1.66	0.83	1.21	9.76	7.92	7.38	7.75	8.13
Big	1.86	1.02	-0.09	0.89	0.23	0.78	12.31	6.21	7.53	8.16	8.21
Avg	1.64	1.19	1.09	1.34	1.00						

Table 5.b

The table reports average monthly excess return on 25 portfolios formed on Size and Operational Profitability. We present results for the three economic block: Latin America, Asia and Eastern Europe. All returns are in US dollars. Data range from january, 2009 to february, 2017. Stocks are sorted into five size portfolios, from small to big, based in their market capitalization. Then, each portfolio is sorted again, into five new portfolios, based on their operational profitability (Ebitda), from low to high. These two sorts produce 25 value-weighted Size-OP portfolios. The table shows the averages of monthly returns in excess of the one-month US Treasury bill rates.

	Mean						Standard Deviation				
	Low	2	3	4	High	Avg	Low	2	3	4	High
Latin America											
Small	1.09	1.26	1.04	0.18	0.35	0.78	10.24	11.41	12.71	12.93	7.33
2	0.01	0.18	0.31	-0.08	0.00	0.09	9.04	10.26	12.11	9.91	6.08
3	0.14	0.34	0.76	0.47	0.56	0.46	8.10	8.51	8.62	6.99	6.41
4	0.12	0.15	0.09	0.11	-0.06	0.09	8.56	6.67	7.13	7.50	5.21
Big	-0.19	-0.23	-0.32	-0.18	-0.31	-0.25	8.42	8.01	4.55	4.98	4.46
Avg	0.24	0.34	0.38	0.10	0.11						
Asia											
Small	1.75	1.67	1.19	1.46	1.65	1.54	6.70	6.44	6.27	6.25	6.50
2	0.97	1.02	0.96	1.02	0.80	0.95	5.96	5.60	5.63	5.65	5.60
3	0.63	0.88	0.85	0.63	0.85	0.77	5.18	5.14	5.33	5.30	5.21
4	0.77	0.93	0.84	1.03	0.72	0.86	6.07	6.00	5.99	5.88	5.87
Big	0.30	0.27	0.17	0.18	0.38	0.26	6.46	6.07	5.37	4.18	4.69
Avg	0.89	0.95	0.80	0.86	0.88						
Eastern Europe											
Small	0.82	0.64	0.37	1.11	0.69	0.73	6.31	6.75	6.98	7.34	6.51
2	0.46	1.05	0.17	0.39	0.08	0.43	6.17	10.96	6.05	7.82	6.05
3	0.15	0.38	0.17	0.47	0.13	0.26	6.56	6.09	6.31	7.48	6.17
4	0.12	0.68	0.47	-0.14	0.58	0.34	6.84	5.88	6.08	5.79	6.37
Big	1.26	-0.34	-0.30	-0.28	0.20	0.11	19.15	6.00	5.72	6.57	6.13
Avg	0.56	0.48	0.18	0.31	0.33						

Table 5.c

The table reports average monthly excess return on 25 portfolios formed on Size and Investment. We present results for the three economic block: Latin America, Asia and Eastern Europe. All returns are in US dollars. Data range from january, 2009 to february, 2017. Stocks are sorted into five size portfolios, from small to big, based in their market capitalization. Then, each portfolio is sorted again, into five new portfolios, based on their investment level, from low to high - conservative to agressive. These two sorts produce 25 value-weighted Size-Inv portfolios. The table shows the averages of monthly returns in excess of the one-month US Treasury bill rates.

	Mean						Standard Deviation				
	Low	2	3	4	High	Avg	Low	2	3	4	High
Latin America											
Small	1.21	0.44	0.54	1.05	0.45	0.74	8.02	9.19	11.88	9.98	11.95
2	0.77	0.19	-0.14	0.10	-0.05	0.17	10.13	8.67	9.21	9.70	8.91
3	0.52	0.36	0.23	0.52	0.40	0.41	7.48	6.68	6.37	9.55	7.32
4	0.09	-0.12	-0.23	0.76	-0.35	0.03	6.23	5.68	8.14	12.23	6.00
Big	-0.07	-0.11	-0.26	-0.15	-0.34	-0.19	6.81	5.80	7.80	7.36	6.35
Avg	0.50	0.15	0.03	0.46	0.02						
Asia											
Small	1.33	1.33	1.69	1.40	1.65	1.48	6.38	6.27	6.12	6.28	6.07
2	0.98	0.84	0.95	0.87	0.87	0.90	5.59	5.67	5.86	5.51	5.60
3	0.90	0.58	0.90	0.80	0.93	0.82	5.22	5.29	5.18	4.98	5.17
4	0.92	0.80	0.86	0.71	0.86	0.83	6.23	6.00	6.25	6.27	6.00
Big	0.18	0.30	0.21	0.24	0.25	0.24	4.96	4.97	4.79	4.59	5.72
Avg	0.86	0.77	0.92	0.80	0.91						
Eastern Europe											
Small	0.69	0.31	0.66	0.45	0.38	0.50	6.77	6.16	6.97	5.51	6.79
2	0.68	0.40	0.25	0.37	0.13	0.37	7.73	8.50	6.63	6.31	6.42
3	0.10	0.25	0.30	0.09	0.85	0.32	6.23	6.07	6.48	6.34	6.95
4	0.09	0.59	0.19	0.34	3.69	0.98	5.75	5.98	6.01	5.97	37.61
Big	0.69	-0.45	-0.45	-0.53	-0.35	-0.22	6.19	5.73	5.93	6.19	6.52
Avg	0.45	0.22	0.19	0.15	0.94						

3.5
Model Performance

In the LHS of our regressions, we use the three sets of 25 portfolios for each economic block, which we described in Section 4.2 and Table 5. For the RHS, we use the factors Rm-Rf, SMB, HML, RMW, and CMA, which we described in Section 4.1 and Table 3. We performed regressions with 2x3 and 2x2x2x2 factor constructions. Because most of the results are not sensitive to the way we build the factors, we comment only the 2x3 sort. The 2x3 portfolios give us a better understanding of how the models capture each of the three measures of interest: value, OP, and Inv.

As in Fama and French (2015), we are less interested in whether a model is rejected than in their relative performance, which we judge using GRS and other statistics. We want to find the model that shows the best fit for the excess returns of portfolios sorted in different ways. We are interested in improvements to the description of the portfolios' average returns provided by adding OP and Inv factors to the original FF three-factor model. Thus, we look mainly to the GRS statistics and the average absolute alpha: $A|\alpha_i|$.

For LA, the GRS statistics are very close for all tested models when we use Size-B/M portfolios and they do not reject the null that all alphas are equal to zero. We cannot see a difference, even for the original CAPM model. The portfolios seem not to be sensitive to these factors, but we notice a reduction at $A|\alpha_i|$ from 37 basis points (bp) using CAPM, to 28 bp using the three-factor model, and then to 26 bp using the five-factor model. When we use Size-OP portfolios as the LHS of the equations, we see a slight improvement when adding more factors to CAPM. Table 6.a shows a GRS going from 0.92 to 0.74 for the four-factor model, and 0.80 for the five-factor model. We notice a pattern that repeats itself for all three blocks: the GRS and $A|\alpha_i|$ for the four- and five-factor models are very close; for some of the results, the four-factor model presents a statistic smaller than that for the five-factor model. It seems to signal a redundant effect between HML and RMW and CMA. Fama and French (2015) report the same result for US data, and equation (3) easily explains this effect. When we use the Size-Inv portfolios, the GRS goes from 1.23 to 1.03 and 1.08 to for the four- and five-factor models, respectively. This result indicates that investors are overlooking financial ratios and demanding a risk premium for other common risk factors that the models do not represent. Indeed, most LA countries have a delicate economic and political situation, which makes financial players cautions in investing, no matter what stocks' financial ratios indicate. Additionally, LA has the poorer sample in the sense of diversified portfolios, with only half of the stocks of EE and less than 20% of Asia, on average. This lack of diversification surely disturbs how the models perform. However, even with the difficulty in fitting to the LA data, growing the model from three to five factors improves its abilities to explain the cross section of returns, as we can see in the reduced pricing error and the GRS statistics for two out of the three sets of LHS. This kind of improvement appears in the literature on developed markets.

For Asia, using the Size B/M portfolios, Table 6.a presents results closer to those presented by Fama and French (2015). All models are incomplete descriptions of the portfolio returns, but we can see a clear reduction in GRS and $A|\alpha_i|$ as we incorporate new factors. We also see that for the four- and five-factor models, both statistics are almost the same, again signaling redundancy between the models. For Size-OP portfolios, we do not see the same effect on GRS, but we have a dramatic reduction in $A|\alpha_i|$ from 0.66 to 0.18, a reduction of 48 bp. Using the Size-Inv portfolios, we find a GRS reduction and, again, a strong reduction in $A|\alpha_i|$.

For EE, for all three types of portfolios used as the LHS of the equations, we see a reduction in the GRS statistics. None of the models can fully explain the excess returns on the Size-B/M portfolios. Table 6.a shows a decreasing GRS from 2.08 for CAPM to 1.92 for the three-factor model, and reaching 1.85 for the five-factor model. The average $A|\alpha_i|$ also presents a clear reduction, from 0.69 to 0.42, and, for the four-factor model, it reaches 35 bp. Using the Size-OP and Size-Inv portfolios, we find the same results: reductions for both GRS and $A|\alpha_i|$. For all three types of portfolios, the GRS statistics for the four- and five-factor models are very close and smaller than the other two models.

Relative to the three-factor model, almost all four- and five-models produce improvements in the average absolute intercept. The only exception is for the LA Size-Inv portfolios, which present a slight increase. The five-factor model produces the biggest improvement in explaining the Size-OP portfolio's excess returns from Asia—22 bp.

Table 6.a shows a measure of the proportion of the portfolio's excess returns left unexplained by the competing models. The numerator, $A|\alpha_i|$, is the average absolute value of the intercepts produced by each model for a given set of LHS portfolios. This value represents the excess returns of the portfolios not captured by a model's factors. The denominator, $A|r_i|$, is the average absolute dispersion of the portfolio's excess returns around the cross-section average excess returns of all 25 portfolios. Therefore, the measure $A|\alpha_i|/A|r_i|$ gives us an idea of how much of the dispersion of the average returns around their cross-section means is unexplained. The results for $A|\alpha_i|/A|r_i|$, for the five-factor model, ranges from 0.53 to 1.08 for all three sets of portfolios for all economic blocks. We conclude that, measured in units of return, the five-factor model leaves an average of 81% of the dispersion of the excess returns unexplained. The dispersion of the excess returns left unexplained by the three-factor model is higher, ranging from 94% to 129%; the CAPM $A|\alpha_i|/A|r_i|$ goes from 103% to 209%. For comparison, the same results for US data reported in Fama and French (2015) are 42-54% for the five-factor model, 54-68% for the three-factor model, and 126-155% for the CAPM. The emerging

markets proportion of unexplained dispersion is higher, as expected, but follows the same pattern as those for US data.

Table 6.a

Summary statistics for tests on CAPM, three-, four- and five-factors models.The table reports the regression results for each model. We try to explain monthly excess returns on 25 Size-BM portfolios (Panel A), 25 Size-OP portolios (Panel B) and 25 Size_Inv portfolios (Panel C). All sets of 25 portfolios are from emerging markets regions. The emerging regions are Latin America, Asia and Eastern Europe. For each set of 25 regressions the table shows the factors that augment RM - RF and SMB. For each regions we used two methods to calculate the factors: 2x3 and 2x2x2x2. Both methods are detailed at the main text. As an evaluation of the models, the table shows the GRS statistic testing whether the expected values of all the 25 intercept estimates are zero. GRS-cdf refers to the cumulative distribution function. $A|\alpha_i|$ and R^2 refer to the average absolut value of intercepts and R^2s. $S(\alpha)$ refers to the average regression intercepts standard deviations and $A|\alpha_i| / A|r_i|$ refer to the average absolute value of the intercept α_i over the average absolute value of r_i , which is the average return on portfolio i minus the average of the portfolio returns. The dataset goes from january 2009 to february 2017, for a total of 98 months.

	2 x 3 Factors						2 x 2 x 2 x 2 Factors					
	GRS	GRS - cdf	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2	GRS	GRS - cdf	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Latin America							**Latin America**					
Panel A: Size-BM Portfolios												
CAPM	0.80	0.27	0.37	0.42	1.31	0.57	0.80	0.27	0.37	0.42	1.31	0.57
HML	0.80	0.27	0.28	0.37	0.98	0.84	0.81	0.28	0.28	0.38	0.98	0.84
RMW CMA	0.90	0.40	0.27	0.36	0.95	0.83	0.77	0.23	0.26	0.54	0.91	0.84
HML RMW CMA	0.89	0.38	0.26	0.36	0.90	0.86	0.75	0.22	0.26	0.34	0.91	0.85
Panel B: Size-OP Portfolios												
CAPM	0.92	0.42	0.34	0.44	1.06	0.57	0.92	0.42	0.34	0.44	1.06	0.57
HML	0.86	0.35	0.32	0.36	0.98	0.80	0.93	0.43	0.30	0.36	0.92	0.81
RMW CMA	0.74	0.20	0.25	0.37	0.78	0.83	0.95	0.46	0.26	0.36	0.81	0.83
HML RMW CMA	0.80	0.28	0.27	0.37	0.82	0.84	0.97	0.48	0.26	0.36	0.81	0.83
Panel C: Size-Inv Portfolios												
CAPM	1.23	0.76	0.38	0.44	1.03	0.56	1.23	0.76	0.37	0.43	1.03	0.57
HML	1.16	0.69	0.25	0.31	0.68	0.84	1.25	0.77	0.24	0.31	0.66	0.85
RMW CMA	1.03	0.56	0.31	0.38	0.84	0.86	0.99	0.51	0.25	0.31	0.68	0.86
HML RMW CMA	1.08	0.61	0.31	0.37	0.87	0.87	1.06	0.59	0.25	0.31	0.68	0.86
Asia							**Asia**					
Panel A: Size-BM Portfolios												
CAPM	2.21	1.00	0.62	0.51	1.97	0.65	2.21	1.00	0.62	0.51	1.97	0.65
HML	2.05	0.99	0.37	0.40	1.17	0.81	2.02	0.99	0.40	0.42	1.26	0.78
RMW CMA	1.74	0.96	0.29	0.36	0.92	0.80	1.63	0.94	0.39	0.41	1.24	0.76
HML RMW CMA	1.75	0.96	0.29	0.35	0.90	0.84	1.61	0.94	0.41	0.42	1.32	0.80
Panel B: Size-OP Portfolios												
CAPM	1.40	0.87	0.66	0.51	2.09	0.61	1.41	0.86	0.66	0.51	2.09	0.61
HML	1.42	0.87	0.40	0.38	1.29	0.77	1.37	0.85	0.48	0.41	1.52	0.74
RMW CMA	1.42	0.87	0.20	0.22	0.60	0.82	2.31	1.00	0.48	0.47	1.52	0.74
HML RMW CMA	1.40	0.86	0.18	0.23	0.57	0.84	2.35	1.00	0.50	0.47	1.60	0.77
Panel C: Size-Inv Portfolios												
CAPM	1.45	0.89	0.61	0.49	1.91	0.70	1.45	0.89	0.61	0.49	1.91	0.70
HML	1.33	0.82	0.30	0.29	0.97	0.83	1.37	0.86	0.37	0.31	1.17	0.79
RMW CMA	1.09	0.62	0.17	0.21	0.54	0.84	1.39	0.86	0.38	0.35	1.19	0.79
HML RMW CMA	1.11	0.64	0.17	0.21	0.53	0.86	1.38	0.85	0.41	0.35	1.29	0.81
Eastern Europe							**Eastern Europe**					
Panel A: Size-BM Portfolios												
CAPM	2.08	0.99	0.69	0.55	1.79	0.60	2.08	0.99	0.69	0.55	1.79	0.60
HML	1.92	0.98	0.43	0.48	1.11	0.72	1.93	0.98	0.37	0.51	0.96	0.76
RMW CMA	1.88	0.98	0.35	0.43	0.90	0.72	2.11	0.99	0.40	0.48	1.03	0.76
HML RMW CMA	1.85	0.97	0.42	0.46	1.08	0.76	2.07	0.99	0.36	0.46	0.94	0.78
Panel B: Size-OP Portfolios												
CAPM	0.94	0.45	0.65	0.52	1.52	0.54	0.94	0.45	0.65	0.52	1.52	0.54
HML	0.88	0.38	0.40	0.46	0.94	0.68	0.88	0.36	0.37	0.47	0.86	0.69
RMW CMA	0.84	0.32	0.33	0.38	0.78	0.69	0.87	0.36	0.39	0.48	0.92	0.71
HML RMW CMA	0.85	0.33	0.37	0.39	0.87	0.73	0.84	0.32	0.35	0.41	0.81	0.73
Panel C: Size-Inv Portfolios												
CAPM	1.32	0.82	0.68	0.73	1.45	0.60	1.32	0.82	0.68	0.73	1.45	0.60
HML	1.20	0.73	0.45	0.91	0.96	0.71	1.20	0.73	0.43	0.83	0.90	0.74
RMW CMA	1.19	0.72	0.34	0.49	0.72	0.74	1.26	0.78	0.45	0.98	0.96	0.76
HML RMW CMA	1.16	0.70	0.37	0.53	0.79	0.77	1.20	0.73	0.37	0.67	0.80	0.77

Table 6.b

Summary statistics for tests on CAPM, three-, four- and five-factors models.The table reports the regression results for each model. We try to explain monthly excess returns on 25 Size-BM portfolios (Panel A), 25 Size-OP portolios (Panel B) and 25 Size_Inv portfolios (Panel C). All sets of 25 portfolios are from emerging markets regions. The emerging regions are Latin America, Asia and Eastern Europe. For each set of 25 regressions the table shows the factors that augment RM - RF and SMB. For each regions we used two different sets of factors: US and Global Developed factors. Both sets are from Keneth French's website. As an evaluation of the models, the table shows the GRS statistic testing whether the expected values of all the 25 intercept estimates are zero. GRS-cdf refers to the cumulative distribution function. $A|\alpha|$ and R^2 refer to the average absolut value of intercepts and R^2s. $S(\alpha)$ refers to the average regression intercepts standard deviations and $A|\alpha_i| / A|r_i|$ refer to the average absolute value of the intercept α_i over the average absolute value of r_i, which is the average return on portfolio i minus the average of the portfolio returns. The dataset goes from january 2009 to february 2017, for a total of 98 months.

	US Factors						Global Developed Factors					
	GRS	GRS - cdf	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2	GRS	GRS - cdf	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
	Latin America						**Latin America**					
Panel A: Size-BM Portfolios												
CAPM	0.99	0.51	0.37	0.52	1.30	0.19	0.92	0.42	0.35	0.51	1.24	0.23
HML	0.97	0.48	0.44	0.48	1.54	0.22	0.84	0.32	0.30	0.43	1.04	0.26
RMW CMA	1.19	0.72	0.46	0.59	1.61	0.25	1.04	0.57	0.52	0.51	1.81	0.30
HML RMW CMA	1.11	0.65	0.47	0.58	1.63	0.26	1.09	0.62	0.52	0.51	1.82	0.31
Panel B: Size-OP Portfolios												
CAPM	1.02	0.54	0.45	0.55	1.38	0.20	0.96	0.48	0.39	0.53	1.19	0.24
HML	1.02	0.54	0.52	0.50	1.60	0.23	0.87	0.36	0.37	0.46	1.14	0.26
RMW CMA	1.11	0.64	0.53	0.59	1.64	0.26	0.86	0.34	0.54	0.47	1.67	0.29
HML RMW CMA	1.00	0.52	0.49	0.57	1.52	0.26	1.07	0.60	0.54	0.47	1.67	0.31
Panel C: Size-Inv Portfolios												
CAPM	1.30	0.81	0.47	0.55	1.28	0.19	1.13	0.67	0.44	0.54	1.19	0.23
HML	1.23	0.75	0.49	0.49	1.35	0.22	1.07	0.60	0.40	0.47	1.10	0.26
RMW CMA	1.32	0.82	0.53	0.59	1.43	0.25	1.16	0.69	0.54	0.52	1.47	0.29
HML RMW CMA	1.22	0.74	0.51	0.58	1.40	0.26	1.17	0.71	0.54	0.52	1.48	0.30
	Asia						**Asia**					
Panel A: Size-BM Portfolios												
CAPM	2.06	0.99	0.75	0.51	2.37	0.19	2.16	0.99	0.84	0.48	2.64	0.26
HML	2.11	0.99	0.59	0.48	1.85	0.25	2.16	0.99	0.71	0.31	2.21	0.31
RMW CMA	2.02	0.99	0.71	0.50	2.23	0.29	2.01	0.99	0.71	0.49	2.25	0.37
HML RMW CMA	2.16	0.99	0.71	0.51	2.23	0.29	1.99	0.97	0.71	0.49	2.25	0.38
Panel B: Size-OP Portfolios												
CAPM	1.67	0.95	0.82	0.51	2.59	0.18	1.58	0.93	0.91	0.48	2.88	0.24
HML	1.58	0.93	0.59	0.46	1.87	0.25	1.61	0.94	0.77	0.48	2.42	0.28
RMW CMA	1.62	0.94	0.77	0.50	2.42	0.28	2.02	0.99	0.83	0.51	2.63	0.35
HML RMW CMA	1.56	0.92	0.73	0.49	2.30	0.29	2.05	0.99	0.83	0.51	2.63	0.36
Panel C: Size-Inv Portfolios												
CAPM	1.39	0.86	0.75	0.46	2.37	0.20	1.46	0.89	0.85	0.43	2.67	0.27
HML	1.49	0.90	0.56	0.42	1.76	0.26	1.61	0.94	0.69	0.44	2.19	0.31
RMW CMA	1.36	0.84	0.71	0.45	2.22	0.31	1.67	0.95	0.72	0.41	2.27	0.39
HML RMW CMA	1.53	0.92	0.68	0.45	2.15	0.30	1.65	0.95	0.72	0.41	2.27	0.39
	Eastern Europe						**Eastern Europe**					
Panel A: Size-BM Portfolios												
CAPM	1.77	0.97	0.42	0.57	1.10	0.28	1.80	0.97	0.43	0.56	1.11	0.38
HML	1.68	0.95	0.44	0.57	1.13	0.31	1.69	0.96	0.38	0.51	0.98	0.39
RMW CMA	2.06	0.99	0.54	0.73	1.40	0.33	1.76	0.97	0.47	0.64	1.21	0.43
HML RMW CMA	1.91	0.98	0.53	0.72	1.38	0.34	1.93	0.98	0.47	0.65	1.21	0.46
Panel B: Size-OP Portfolios												
CAPM	0.97	0.49	0.49	0.63	1.14	0.25	0.93	0.43	0.45	0.58	1.06	0.34
HML	0.97	0.48	0.51	0.64	1.19	0.24	0.90	0.40	0.41	0.52	0.96	0.37
RMW CMA	1.12	0.66	0.55	0.74	1.29	0.29	0.99	0.51	0.48	0.62	1.12	0.41
HML RMW CMA	1.04	0.57	0.56	0.74	1.32	0.30	1.01	0.54	0.48	0.61	1.12	0.43
Panel C: Size-Inv Portfolios												
CAPM	1.48	0.90	0.53	0.11	1.13	0.29	1.38	0.86	0.49	0.99	1.05	0.38
HML	1.58	0.93	0.58	1.14	1.23	0.31	1.42	0.87	0.48	0.92	1.01	0.39
RMW CMA	1.66	0.95	0.63	1.31	1.34	0.34	2.06	0.99	0.55	0.87	1.17	0.44
HML RMW CMA	1.81	0.97	0.63	1.32	1.34	0.36	2.04	0.99	0.56	0.87	1.18	0.46

Table 6.b reproduces the same tests reported in Table 6.a, but using US and global developed factors from Kenneth French’s website. The GRS for all four models are almost the same, and in some cases, the CAPM has smaller statistics than the other models do. This means that including new explanatory factors does not help to explain the excess returns for all three sets of portfolios.

If we compare Table 6.a with 6.b, we see that all statistics worsened when we exchange local factors with US and global factors. The GRS increases, the $A|\alpha_i|$ and $A|\alpha_i|/A|r_i|$ are substantially bigger, and the R^2, for all models, shows a dramatic reduction. These findings lead us to conclude that emerging market economies are segmented from the US and global economies. Again, as we notice that for local factors, many of the models tested are rejected for both local and US/global factors. However, we are looking more for their relative performance than for whether they are accepted or rejected. When the models are not rejected using local factors, the lack of evidence is stronger, and when they are rejected, the rejections are weaker for local factors than for US and global factors.

3.6
Final Discussion

With the growing integration of world stock markets in the last decades, the importance of emerging markets as a field of research constantly rises. There are many challenges in understanding this environment. Economic vulnerability, political instability, small samples, and a low degree of diversification are some of the issues that disturb the regular functioning of capital markets, making it harder to understand the real forces behind stocks' returns. However, we must try to overcome these difficulties and learn how emerging markets work and how international integration occurs. This study is an effort to reveal more evidence on in this respect using data from 12 emerging countries divided into LA, Asia, and EE economic blocks. We present results for how size, value, profitability, and investment affects the excess returns on stock portfolios after the 2008 global economic crisis.

This paper makes four contributions to the literature. First, we divide stocks into 25 portfolios based on Size and B/M, Size and OP, and Size and Inv ratios. With these portfolios in hand, we try to explain their excess returns with linear regression. We use the CAPM and FF three-, four- and five-factor models. We find clear evidence that the increase in the RHS of the equations from the CAPM and the FF models, improves the fit of the model. Second, we find that the fit of the four- and five-factor models are very close in almost all cases, which seems to be evidence of a redundant effect between HML (high minus low) and RMW (robust minus weak) and CMA (conservative minus aggressive). This result is similar to those in Fama and French (2015). The evidence suggests that a four-factor model that drops the HML performs as well as the five-factors model, or slightly better. Third, we show a clear size effect on excess stock market returns, and few evidence of the other effects. We do not find that value and profitability have clear effects and find some slight effect of investment. These results are not consistent with those reported for developed markets. However, they are not surprising either, considering all of the economic and political problems that most of these countries face and the low number of liquid stocks in most of its markets, which makes for a suboptimal diversification in its portfolios. Finally, as a fourth contribution, we find that local factors perform better than US and global factors in explaining the returns of diversified portfolios, showing evidence of emerging market segmentation.

From the four results above, three are similar to those found for the US and developed markets, and one is partially different. This encourages us to say that we find results and patterns that, if not as strong and clear as those for developed markets, show evidence that they follow the same trends.

Our similar results show that the fourth and fifth factors help to explain portfolio excess returns, reducing its unexplained dispersion. The value factor seems somewhat redundant in the presence of the profitability and investment factors, which is exactly what we see for US data (Fama and French, 2015). Finally, the local factors better explain portfolio excess returns than for the US and global factors. Griffin (2002) also reports this pattern of market segmentation for developed countries. Hou, Karolyi, and Kho (2011) also document segmentation in developed markets (global x local factors) and propose a combination of local and foreign factors, called an international version of the model, to overcome this segmentation.

The main difference between our results and those found in the literature for developed markets is related to the patterns of factors' average returns. We present a clear size effect that follows exactly the same return pattern in both emerging and developed countries. The same effect does not appear in the value and profitability factors, and appears partially for the investment factor. It is important to highlight that although the patterns in the risk factors' average returns are not all the same for emerging and developed countries, they act together in the model in the same way for both kinds of economic blocks; that is, the five-factor model explains the cross-section of returns better than the three-factor model does either for developed and emerging markets.

The main implications of our results are that they illustrate that the five-factor model proposed to explain portfolio excess returns from developed economies also works better than the three-factor model for emerging economies. The results indicate which risk factors were relevant for investors in the three main emerging economic regions that host almost half of the world's population. Our results are important because they show that emerging economy investors are pricing assets, to a certain extent, with a similar rationality as investors in developed economies do. This means that these markets are getting closer to developed markets in the sense that other disturbances, such as political and social issues, seem to be growing less relevant to investors than financial indicators are. It is one signal of the growing maturity of their economies and act as an incentive for international investments.

4 The Fama-French’s Five-Factor Model Relation with Interest Rates and Macro Variables

4.1 Introduction

Since the seminal series of Fama and French (FF hereafter) articles from 1992 to 2015, exploring factor models that explain the cross section of portfolios returns, an enthusiastic debate has took place about the economic meaning of the proposed factors. FF identify some common risk factors in stocks returns. Their first proposition to address this issue is the well-known three factor model, where the first one is an overall market factor, i.e., the excess market return ($R_M – R_f$), the second reflects the size of the firm (SMB) and the last factor reflects the book-to-market equity effect – value x growth stocks (HML).

FF (1996) suggests that we can interpret their factors as proxies for state variables, whose innovations describe the investment opportunity set. For US data, the market factor usually has a slope approximately equal for all 25 size/book-to-market portfolios. Therefore, it is interpreted as representing the risk of being a stock instead of a risk-free asset. Market factor average return captures general stock-market risk, which affects equally almost all companies. The slope on SMB drops from small-size companies to large-size companies as it is the spread of the returns due to size-related risk factor. This factor, with this interpretation, confirms the evidence on Huberman and Kandel (1987) of variations in stock returns relative to the size of the companies, which are not captured by the market return. The authors interpret the HML factor as a proxy for profitability. Weak firms, with long periods of low profits tend to have high book-to-market and positive loading on HML. Investors demand higher expected returns to buy these stocks. The opposite happens to strong firms, with long periods of high profits. There was already evidence of this on Chan and Chen (1991). Chen and Zhang (1998) present a similar result some years after. Later on, FF show that HML's ability to explain expected returns might be complemented, because the current value of the stock is also correlated with operational profitability and investment (Fama and French, 2015).

Several other interpretations were made in the last decade or so, and most of it provides support to the risk-based explanation. Jagannathan and Wang (1996) relate the FF factor to business cycles using a conditional version of CAPM, which resembles the multi-factor model of Ross (1976), and show that when the conditional version of CAPM holds, a two-factor model obtains unconditionality They use the spread of corporate bonds as a complement to market return in predicting future economic conditions. Lettau and Ludvigson (2001) and Vassalou (2003) show that the presence of measures of macroeconomic risk reduces the explanatory power of SMB and HML. Petkova (2006) include shocks to the aggregate dividend yield and term spread, default spread, and one-month Treasury-Bill rate and show they are proxies for HML and SMB factors. She shows that a model with innovations to these variables perform better than the original three-factor model in explaining the cross section of portfolios returns. Gulen, Xing and Zhang (2010) study the flexibility of value and growth companies in adapting to bad economic conditions. Lioui and Poncet (2011) revisited Petkova (2006) and show that the econometric procedure to exclude innovations in macroeconomic variables may influence the significance of the other factors. More specifically, the order of the factors and variables to the orthogonalization procedure may influence the results. In this articles we show that although this concept is correct, it does not invalidate Petkova's (2006) results. Actually, we use this fact to show that FF factors do proxy for macroeconomic variables, as proposed by Petkova. Finally, Fama and French (2015) present a five-factor model, which includes two new factors related do the firm's market value and, consequently, to HML. The first one, RMW is the difference between the returns on portfolios of firms with robust and weak operational profitability. The second, CMA, is the difference between the returns on portfolios of firms with low and high investment, which they call conservative and aggressive. We analyze the relation between these two factors with state variables that describe future investment opportunities, following the framework adopted by Campbell (1996) and Petkova (2006), in the context of Fama and Fench (2015) five-factor model.

From this discussion in the literature, we propose the following question: what macroeconomic variables could provide some intuition on the five-factor model of FF (2015)? Both new factors are related to HML. RMW is the spread between portfolios with robust and weak operational profitability and CMA is related to the investment policy adopted by firms. Is there a set of variables that describe future investment opportunities whose innovations can proxy the five factors? Petkova (2006) presents a set of four macroeconomic variables that are proxies for HML and SMB factors. This set of variables cannot explain the RMW factor. Cochrane (2001) alerts that ICAPM framework is not a “fishing license” to add multiple factors to the model. Only variables with power to predict future investment opportunities should be accepted in the model. More than that, we believe we should use variables with some theoretical background to justify their use. Bernard (1986) demonstrates that underlying firm characteristics could create interaction between unexpected inflation and operating profitability. Inspired by this result, we added innovations to CPI to the set of economic variables proposed by Petkova (2006) and show that the RMW factor loses its explanatory capability. More than that, in the presence of the market excess return and the first principal component of the new five-variable set, all other four factors of FF (2015) lose their explanatory ability. We notice than that dividend yield, one-month Treasury-Bill rate and default spread have strong correlations with CPI innovations and yield curve slope do not have significant correlation with the other four variables. Interestingly, CPI and yield curve slope are the main drivers to future changes in monetary policy, so we take all firsts three out of the model. We show then that a model based on excess market return and innovations in the CPI and in the slope of the term structure has a higher explanatory power than the FF five-factor model with a pricing error statistically indistinguishable from zero.

This article is organized as follows. After this introduction we present the data used in the model and following that we discuss the FF five-factor model and innovations approach and develop our model. In section 4 we present the results and in section 5 we conclude.

4.2 Data

In this study, we use monthly data from July 1963 to June 2017, for a total of 648 months. The beginning of the period is set to 1963 to coincide with most of the papers examining FF models, as Fama and French (1992, 1993, 1996), Campbell (1996), Petkova (2006), Lioui and Poncet (2011), Chen and Petkova (2012), Fama and French (2015), among others. We use excess returns on the market portfolio and returns on portfolios build on size (SMB), value (HML), operational profitability (RMW) and investment (CMA) (see, Fama and French 2015). Those time series returns are from Professor Kenneth R. French data library website, as well as the excess returns on 25 portfolios ordered by size and book-to-market. According to Petkova (2006), these 25 portfolios are a benchmark in testing competing models and one of the most challenging set of portfolios in the asset pricing literature. Besides these returns series, we use other five state variables time series throughout our study. These are the dividend yield of the S&P500 value-weighted portfolio; the interest rate term spread, which is the difference between the 10-year and the 1-year Treasury rates; the default spread, which is the difference of a Baa corporate bond yield relative to the yield on the 10-year Treasury; the yield on 1-month T-bill and the CPI. Those variables are used in the context of ICAPM of Merton (1973) that offers a risk-based explanation for the factors in an asset-pricing model, in which the factors might proxy for innovations in state variables that describe changes in the future investment opportunities. The term spread and the 1-month T-bill represent the slope and the level of the term structure, respectively (see, Litterman and Sheinkman 1991). The dividend yield and the spread default represent the first moment of conditional distribution of asset returns (see, Campbell and Shiller, 1988 and Fama and French, 1989). We use the CPI time series as a variable that predicts changes in investment opportunities and interactions with operational profitability (see, Bernard, 1986). This interaction is not represented by the other four state variables described above. The bonds, the default spread and the CPI time series are from FRED® website, the economic database from Federal Reserve Bank of St Louis.

4.3
The Five-Factors Model and the Innovations Approach

4.3.1
Fama and French Modeling

In the well-known three-factor model of Fama and French (1993), the authors propose that the expected return on a portfolio in excess of the risk-free rate is explained by:

$$R_{i,t} = \alpha_i + \beta_{i,M}R_{M,t} + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \varepsilon_{i,t} \quad (1)$$

Where, $R_{M,t}$, SMB_t and HML_t are the excess return on a market portfolio, the size factor (small minus high market cap), the value factor (high minus low book-to-market ratio) and α_i is a bad model specification indicator.

To test this theory, we usually use the Fama-MacBeth (1973) cross-sectional method. As the first pass of this method, we estimate equation (1) multiple time-series regressions that provides asset's loadings with respect to the factors adopted. Then, as a second pass, we estimate equation (2) cross-section regressions to relate the excess returns of all portfolios with their exposure to the risk factors of the model. The γ terms represent the price of risk for each factor.

$$R_{i,t} = \alpha_i + \gamma_{t,M}\beta_{i,M} + \gamma_{t,SMB}\beta_{i,SMB} + \gamma_{t,HML}\beta_{i,HML} + \varepsilon_{i,t} \quad (2)$$

Many studies show evidence that the average returns are correlated with B/M ratio. Firms with high book-to-market tend to have persistent low earnings and positive slopes on HML. Firms with low book-to-market tend to present high earnings and negative slopes on HML (Fama and French, 1996). Fama and French (2015) use the dividend discount model to show that profitability and investment add to the description of average returns provided by B/M. They show, as in Miller and Modigliani (1961), that total market value of a firm at time *t* is:

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau} \quad (3)$$

where, $Y_{t+\tau}$, is the total equity earnings for period $t+\tau$ and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity. Dividing by time *t* book equity we have:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \quad (4)$$

From this equation, they make three statements:

(1) Everything fixed in (4) except M_t and *r*, tells us that a lower M/B implies a higher expected return;

(2) Everything fixed in (4) except Y_t and *r*, tells us that a higher expected earnings implies a higher expected return;

(3) Everything fixed in (4) except dB_t and *r*, tells us that a higher expected growth in book equity – investment – implies a lower expected return;

Those statements led the researchers to examine a model that adds investment and profitability factors to their prior model. Therefore, they add two more factors to the three-factor model:

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,RMW} RMW_t + \beta_{i,CMA} CMA_t + \varepsilon_{i,t} \quad (5)$$

where RMW is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA is the difference between the returns on diversified portfolios of the stocks of low and high investment firms – conservative and aggressive. The remaining factors are just the same as in (1). The second pass for this proposed model is:

$$R_{i,t} = \alpha_i + \gamma_{t,M}\beta_{i,M} + \gamma_{t,SMB}\beta_{i,SMB} + \gamma_{t,HML}\beta_{i,HML} + \gamma_{t,RMW}\beta_{i,RMW} + \gamma_{t,CMA}\beta_{i,CMA} + \varepsilon_{i,t} \quad (6)$$

If the asset's loading related to each specific risk factor is relevant in determining the asset returns, then the respective γ should be statistically significant. If the model specification is good, then the α term should be statistically indistinguishable from zero.

To conduct our study we use a complete set of five state variable, dividend yield, term spread, default spread, one-month T-Bill rate and CPI. All these variables are described at section 2. Our framework is the five-factor model of Fama and French (2015). We adopted a VAR approach described in Campbell (1996), Petkova (2006) and Chen and Petkvoa (2012) to extract the innovations in each state variable and use them, plus the market excess return, as factors in the model. We tested several combinations of state variables with and without the original FF factors. Basically we assume the general model for unconditional expected excess returns on stock portfolios:

$$R_{it} = \alpha_i + \gamma_{t,M}\beta_{i,M} + \sum \gamma_{t,K}\beta_{i,K} + \sum \gamma_{t,FF}\beta_{i,FF} + \varepsilon_{it} \quad (7)$$

where, $\gamma_{t,M}$, $\gamma_{t,K}$ and $\gamma_{t,FF}$ are the Market risk premium, the price of risk for the innovations in state variables and the price of risk for the FF factors (SMB, HML, RMW and CMA), respectively. The betas are the loadings on Market returns, innovations to the state variables and FF factors, estimated from the general return-generating process:

$$R_{it} = \alpha_i + \beta_{i,M}R_{M,t} + \sum \beta_{i,K}\, e_{k,t} + \sum \beta_{i,FF}\, e_{FF,t} + \varepsilon_{it} \quad (8)$$

where $R_{M,t}$ is the return on Market portfolio in excess of the risk-free rate and $e_{k,t}$ and $e_{FF,t}$ are the innovations to the state variable K and to the Fama and French factors, respectively, at the end of period *t*. The unexpected variance of the factors should command a risk premium, according to the asset pricing literature. Here we adopted the innovations in FF factors following Petkova's (2006) approach, as the returns on factors and their innovations have a high correlation, ranging from 88% to 95%. The state variables and FF factors' innovations are estimated through a VAR approach that describes theirs time series dynamics, as we illustrate below. To check the regression model validity, we compute the adjusted cross-sectional R^2, proposed by Jagannathan and Wang (1996). The gammas are subject to errors-in-variables, as the betas are generated regressors. To work around this issue, the *t-statistics* associated with the γ terms are adjusted, following Shanken (1992).

4.3.2 The VAR approach

To evaluate the model in Equations (7) and (8) it is necessary to estimate the set of state variables innovations. For this task, we adopt the approach described by Campbell (1996), also assumed by Petkova (2006), Lioui and Poncet (2011) and Chen and Petkova (2012) for the time series dynamics of the state variables. A first order VAR was used based on a state vector z_t that contains RM, HML, SMB, RMW, CMA, DY, T, DS, RF and CPI, not all of them at the same time. We make several evaluations of the model and for each one we use a different set of variables, as we show in the next section. The model is then described in matrix form by:

$$z_t = Az_{t-1} + u_t \quad (9)$$

where the residuals will be the innovations used as the risk factor in Equation (8). Campbell (1996) explains that it is very difficult to analyze the result of a VAR if the factors are not orthogonalized and normalized in any way. In the above model, the system was triangulated so that the innovations regarding excess market return are not altered, but the rest of them are orthogonal in relation to those immediately before. Thus, in a model using RM, HML, SMB, DY and T, for example, the innovations on DY are orthogonal to those of excess market return, HML and SML. The same occurs for the innovations on T, relative to the upfront variables. We also normalize the system in a way that the innovations of new factors present the same variance as the excess market return.

To eliminate a possible look-ahead bias, the innovations at time *t* are calculated with information available up to time *t*, that is, the VAR is estimated using only data up to time *t*. The first VAR estimation contains the firsts 36 month of the series. Therefore, the first data for factors' innovations are for July 1966.

4.4
Results

We start this study by estimating the Fama and French (2015) five-factor model, using the two-pass approach of Fama and MacBeth (1973). We use data from Professor French's website. We describe the data at section 2. The original model is the benchmark for our analysis. The authors suggest that we could interpret their factors as proxies for state variables, whose innovations describe the investment opportunity set (see, Fama and French, 1996). Hereafter, we estimate some factor models following Fama and French philosophy, with different set of factors. We use as factors the market's excess return, innovations in five state variables – aggregate dividend yield and term spread, default spread, one-month Treasury-Bill rate and CPI – and innovations in four FF factors. We would like to understand if state variables are proxies for FF factors and the different ability of each model to explain the portfolios' excess returns. We are concerned about how good is the fitting, if the estimated coefficients are statistically significant and if the model's pricing errors are statistically indistinguishable from zero.

For the original five-factor model (Fama and French, 2015) we find an explanatory ability of 72% and the pricing error is strongly significant. Three out of five factors are significant. These results are what we were expecting, based on the literature. We present the summary statistics on Table 1.

Table 1
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed over the full sample in one time-series regression. The model is the Fama and French (2015) five-factor model. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Fama and French Five-Factor Model							
	γ_0	γ_M	γ_{SMB}	γ_{HML}	γ_{RMW}	γ_{CMA}	Adj. R^2
Estimate	1.04	-0.48	0.30	0.34	0.47	-0.03	0.72
SH *t-stat*	3.08	-1.37	2.32	2.83	2.64	-0.16	

Petkova (2006) shows that SMB and HML proxy for innovations in a set of state variables including dividend yield, interest rates term spread, default spread and 1-month T-bill. The dividend yield and the default spread represent the first moment of the conditional distribution of asset returns (see, Campbell and Shiller, 1988 and Fama and French, 1989) and the 1-month T-Bill and the term spread the level and slope of the yield curve (see, Litterman and Sheinkman 1991). When she included the innovation in those macroeconomic variables in the original three-factor model, as described on equation (7), the FF factors lose their explanatory power. She adopted Campbell's (1996) approach to get factors innovations, running the VAR described in 3.2, arranging the factors with the market return and state variables coming first and then the FF factors, so as state variables can capture information before the factors. According to Petkova, a model in which the factors are the market excess return and the innovations on that set of state variables explain the cross section of portfolio returns better than the original three-factor model does.

Although we adopt a similar approach, our paper differs from Petkova (2006) as we use the five-factor model of Fama and French (2015) as our benchmark and use CPI as an addiction the set of state variables representing the risk factors. We add innovations in CPI to the set of state variables as we notice that the original set could not capture the RMW factor ability to explain the portfolios' excess returns. Bernard (1986) demonstrates that there might be a relation between unexpected inflation and operating profitability. Hence, we test if CPI can work as a proxy for the RMW factor and we show that in the presence of innovations in CPI, RMW lose its statistical significance. Moreover, when we adopt a specification that includes the market returns and innovations in CPI and slope of the yield curve, the model fits the data better than the five-factor model. With this specification, we obtain a relationship between the main drivers of the monetary policy management – inflation and yield curve slope – conducted by Central Banks and the cross-section of portfolios excess returns.

Lioui and Poncet (2011) argue that the order of the orthogonalization process could have interfered in Petkova (2006) results. We test for this and confirm that SMB and HML proxy for her set of four state variables. We estimate the VAR described in 3.2 putting FF factor behind the state variables in the factor vector. The factors vector, from equation (9), became:

$$z_{t-1} = [\, RM_{t-1}, Div_{t-1}, Term_{t-1}, Def_{t-1}, RF_{t-1}, SMB_{t-1}, HML_{t-1}] \quad (10)$$

where, RM, Div, Term, Def, RF, SMB and HML are the market excess return, S&P500 dividend yield, term structure slope, default spread, risk free rate and the two factor of FF (1993) model.

We then estimate the model with excess market returns, innovations in the four state variables and SMB and HML, just as Petkova did. We find results similar to hers. With this specification, once we estimate the model, FF factors lose their explanatory power, that is, the four state variables "steal" the information in FF factors innovations, and R^2 reaches 70% (untabulated results).

Then, we estimate the model using only the excess market returns and innovations in the four state variables as factors. The R^2 reaches 72% (untabulated results), that is, removing SMB and HML as factors do not compromised the ability to explain the cross-section of the portfolios returns.

We then, change the order, putting SMB and HML in front of the state variables, so as the factor vector became:

$$z_{t-1} = [\, RM_{t-1}, SMB_{t-1}, HML_{t-1}, Div_{t-1}, Term_{t-1}, Def_{t-1}, RF_{t-1}] \quad (11)$$

We estimate the VAR again and use only the innovations on four state variables to estimate the model. The R^2 falls down to 32% (untabulated results). It means that, in this specification on the model, the factors, SMB and HML, "steal" the information in the variables during the orthogonalization process. As we do not use their innovations in the time-series regressions, we experience a severe reduction in the model's ability to explain portfolios returns. Hence, we may deduce that the set of state variables are proxy for SMB and HML. Depending on which set of factors we put first in the VAR specification, the innovations in state variables explain 72% or 32% of the assets returns. Therefore, SMB and HML proxy for a set of state variables that describes both, the level and the slope of the interest rates and the first moment of the conditional distribution the assets returns. Petkova (2006) present these results only in the first two specifications of the model: the model with excess market returns and innovations in the state variables and SMB and HML and the model with excess market returns and only innovation in the state variables. We test the third specification, that is, if changing the order of the variables in VAR specification would lead to a different conclusion, as proposed by Lioui and Poncet (2011). We find that changing the order of the VAR does influence the result, but in a way that confirms Petkova's (2006) results.

Petkova (2006) presents a set of state variables that proxies for SMB and HML. We go further and ask which set of state variables proxies for SMB, HML RMW and CMA. We repeat all the above procedure with Fama and French five-factor model – including RMW and CMA along with the other FF factors in both factors vectors (10) and (11) – and find similar results.

Table 2 shows the summary statistics for the complete specification, that is, the cross-section regression of the model with market excess return and the innovations in the four state variables and in the four FF factors, described generically in equation (7).

Table 2
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. The model is the Fama and French (2015) five-factor model, augmented with innovations in the aggregate dividend yield and term spread, default spread, one-month Treasury-Bill rate, and Fama and French factors, SMB, HML, RMW and CMA. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

	Model with innovations in four State Variables and five Fama and French Factors										
	γ_0	γ_M	γ_{Div}	γ_{Term}	γ_{Def}	γ_{RF}	γ_{SMB}	γ_{HML}	γ_{RMW}	γ_{CMA}	Adj. R^2
Estimate	1.06	-0.56	1.75	1.92	-0.74	1.47	0.46	0.01	1.85	1.48	0.77
SH *t-stat*	1.97	-0.92	0.95	1.66	-0.70	0.88	1.26	0.02	2.74	1.67	

We can see an R^2 of 77%, which suggests that the model has a good ability to explain the cross-section of portfolio's returns, even better than the original model. Three out of four FF factors, on the other hand, lose their explanatory ability. There are two intriguing results in this model. The first is the fact that RMW is the only FF factor that remains significant in the presence of the innovations in state variables, which leads us to a new state variable that proxies RMW – the operational profitability factor. The second is the low level of significance of all factors, despite of an R^2 of 77%, which leads us to realize that too many factors explaining the same economic underline facts may be causing confusion. We address these two issues in separate steps. To deal with the significance of RMW we add a new state variable: innovations in CPI. To deal with the possible multicollinearity we extract the first Principal Component of the set of state variables.

Before implementing this two procedures, we test two other specifications by changing the order of the VAR factor vector to see if putting FF factor in front or behind the state variables in the factors vector influences the results. We estimate the model using the state variable vector for the VAR system as in (10), adding RMW and CMA by the end of the vector, and we use only the innovations in the state variables to estimate the time-series regressions. The summary statistics are presented at Table 3.

Table 3
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return and the state variables come first and then comes the FF factors. The model includes only the excess market return (M) and innovations in the aggregate dividend yield and term spread, default spread, and one-month Treasury-Bill rate. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innovations in State Variables - VAR w/ FF fcts behind State Variables							
	γ_0	γ_M	γ_{Div}	γ_{Term}	γ_{Def}	γ_{RF}	Adj. R^2
Estimate	0.78	0.08	-1.77	1.88	-0.14	3.29	0.69
SH *t-stat*	1.70	0.18	-1.43	1.43	-0.13	1.85	

Table 3 reports statistical significance levels similar to those in Table 2, and an R^2 in the same rage, almost 70%. The significance of the errors-in-variables adjusted factors' price-of-risk are still lower than we expected. At Table 4 we report the summary statistics for the opposite VAR specification, that is, we arrange the state variables vector with the FF factors coming first and then the state variables, as in (11), adding the RMW and CMA factors in the middle of the vector – between HML and the state variables. Once we estimate the innovations in all factors, we use the unexpected shocks in the state variables to feed the time-series regressions. The most important change is the reduction of the R^2 from 69% to 35%. Therefore, when we put FF factors first, they "steal" the information from the state variables, that is, their innovations are no longer able to explain 69% of the portfolios' returns, but only 35%. Hence, a good amount of information is common among state variables and FF factors that make it reasonable to say that they are proxy for each other. That result is similar of that presented in Petkova (2006) for the three-factor model.

So far, we have shown that there is a correlation among the set of state variables proposed by Petkova (2006) and the five-factors model of Fama and French (2015), although this correlation does not embrace all the factors. Changing the order of the VAR estimation interferes in the results, but in a way that confirms that this set of state variables are a reasonable proxy for the factors. Next, we test for the lack of significance of the state variables prices-of-risk and search for a state variable correlated with the operational profitability.

Table 4

Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return and FF factors come first and then comes the state variables. The model includes only the excess market return (M) and innovations in the aggregate dividend yield and term spread, default spread, and one-month Treasury-Bill rate. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innovations in StateVariables - VAR w/ FF fcts first then State Variables							
	γ_0	γ_M	γ_{Div}	γ_{Term}	γ_{Def}	γ_{RF}	Adj. R^2
Estimate	0.30	-0.04	-1.22	3.54	-0.39	0.69	0.35
SH *t-stat*	0.60	-0.07	-0.74	2.59	-0.35	0.35	

To work around the issue of the lack of significance for almost all factors, we choose to extract the first Principal Component (PC) from the set of four state variables and use it as the input to the VAR estimation. Its innovations are then use to feed the time-series regressions. The idea is to concentrate all explanatory ability from the state variables in only one factor, and test if innovations in this factor proxy for FF factors. Table 5 shows the summary statistics for this test. The PC factor is strongly significant considering Shanken's adjusted for errors-in-variables *t-Statistics*. In the presence of the innovations in the PC of the state variables, three out of four Fama and French factors lose their explanatory ability and become statistically insignificant. Only RMW remains significant. The model's ability in explain the cross-section of portfolios returns reaches 77%, higher than the original Fama and French (2015) model, presented in Table 1, and as high as the model containing the set of four state variables and the four FF factors, presented in Table 2. More than that, in the presence of the first Principal Component of the set of four state variables, the pricing error becomes statistically insignificant, which does not happen to the models presented in Tables 1 and 2. The advantage of using the first Principal Component is that it does not allow for multicollinearity issues among state variables. The first PC of the state variables represents its conjoint variability and Table 5 shows us how much of the portfolios returns they can explain together.

Table 5
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return and the first principal component of state variables come first and then comes the FF factors. The model includes the excess market return (M) and innovations in the first principal component of the state variables (not including CPI) and in the FF four factors, SMB, HML, RMW and CMA. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innivations in four State Variables 1st Principal Component and FF Five-Factors								
	γ_0	γ_M	γ_{4SVPC}	γ_{SMB}	γ_{HML}	γ_{RMW}	γ_{CMA}	Adj. R^2
Estimate	0.46	0.08	-2.31	0.24	0.18	1.60	0.43	0.77
SH *t-stat*	1.26	0.18	-2.32	0.88	0.63	3.23	0.67	

We find that the state variables proposed by Petkova (2006) do not proxy RMW – operating profitability – factor in FF five-factor model (Fama and French, 2015). Bernard (1986) shows that underlying firm characteristics could create interaction between unexpected inflation and operational profitability. Inspired by this result, we propose to add CPI to the set of the original four state variables. We find that in the presence of the first principal component of the four original variables, RMW do not lose its explanatory power, but when we use the first PC of the set including CPI it does lose its statistical significance. We find then a clear correlation between RMW, operating profitability factor, and unanticipated inflation.

Table 6 show the result for the model specification that uses the market excess return and innovations in first PC of five state variables and FF factors to explain portfolios excess returns. If betas with respect to innovations in a factor are important determinants of average returns, then there should be a statistically significant price of risk associated with that factor. When loading on the first PC of the set of five state variables are present in the model, loadings on SMB, HML, RMW and CMA lose their ability to explain the cross-section of portfolios excess returns. Moreover, the adjusted for errors-in-variables price-of-risk for the loading on the first PC is strongly significant, the pricing error is statistically insignificant and the R^2 of the model reaches 78%, the highest above all models tested so far.

Table 6
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return and the first principal component of state variables come first and then comes the FF factors. The model includes the excess market return (M) and innovations in the first principal component of the state variables (including CPI) and in the FF four factors, SMB, HML, RMW and CMA. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innnovations in five State Variables 1st Principal Component and FF Five-Factors								
	γ_0	γ_M	γ_{5SVPC}	γ_{SMB}	γ_{HML}	γ_{RMW}	γ_{CMA}	Adj. R^2
Estimate	0.64	-0.11	3.13	0.36	0.48	0.90	1.29	0.78
SH *t-stat*	1.64	-0.24	2.31	1.22	1.54	1.66	1.58	

We may see that the unexpected shocks on the set of state variables proxy for the information in the FF factors. We then test the model without FF factors, using as factors the market excess return and innovations in the five state variables, named, aggregate dividend yield and yield curve's slope, default spread, one-month Treasury-Bill rate and CPI. Table 7 show the summary statistics for this specification.

We find a strong ability in explain the portfolios returns of 76% and a pricing error statistically insignificant, but the price-of risk for the factors are no longer significant again. When we use the first principal component of the state variables, it commands a strongly significant price-of-risk, but the variables all together are once again, presenting some sign of multicollinearity. Examining the data, we find that dividend yield, default spread and 1-month T-Bill have a relevant degree of correlation with CPI, with correlations ranging from approximately 58% up to 66%, and have low correlation with term structure slope. Actually, the term structure slope has low correlation with all other variables. Therefore, we test if a model in which the factors are only the excess market return and the unexpected shocks to CPI and yield curve slope perform better than the above models.

Table 7
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return and the state variables come first and then comes the FF factors. The model includes only the excess market return (M) and innovations in the aggregate dividend yield and term spread, default spread, one-month Treasury-Bill rate and CPI. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innovations in five State Variables								
	γ_0	γ_M	γ_{Div}	γ_{Term}	γ_{Def}	γ_{RF}	γ_{CPI}	Adj. R^2
Estimate	0.70	-0.14	-1.47	1.83	-0.23	1.32	-1.27	0.76
SH *t-stat*	1.78	-0.33	-1.34	1.58	-0.25	0.86	-1.34	

This is a particularly interesting specification of the model, as these two state variables are the main indicators of the execution of the monetary policy by Central Banks. A new contribution of this paper is to provide a link between the variance of the excess returns of the 25 Fama-French portfolios and the macro economic variables that indicates the future expectations about changing in monetary policy guidelines.

Table 8
Summary statistics for Fama-MacBeth cross-sectional regressions using the excess returns of 25 portfolios sorted by size and book-to-market. The factor betas, which are the independent variables in the regressions, are computed up to each time *t*, starting with the firsts 36 months, with several time-series regressions. In the VAR used to calculate the innovations, the market return, CPI and the term structure slope come first and then comes the FF factors. The model includes the excess market return (M) and innovations in the CPI and the term structure slope. The adjusted R^2 follows Jagannathan and Wang (1996). The *t-statistics* are adjusted for errors-in-variables, presented as SH *t-stat*, following Shanken (1992). The sample period is from July 1966 to June 2017, 648 months.

Model with innovations in CPI and Term Structure 2^{nd} PC					
	γ_0	γ_M	γ_{Term}	γ_{CPI}	Adj. R^2
Estimate	0.50	0.10	-2.37	-2.32	0.73
SH *t-stat*	1.09	0.20	-2.08	-1.68	

As we can see in Table 8, once we use as model's factors the market excess returns and the innovations in CPI and in the slope of the yield curve, we get an R^2 of 73%, which is better than the 72% of the original model, presented in Table 1.

We get the pricing error statistically insignificant and we get the price-of-risk for the CPI significant for errors-in-variables adjusted *t-statistic* at 10% significance level, and the price-of-risk for the slope of the yield curve significant at 5% significance level. We also see at Table 8 an inverse relation among portfolio excess returns and innovations in CPI and yield curve slope. The negative sign is aligned with our theoretical perception. Ammer (1994) documents the relation between inflation and stock returns in ten industrialized countries and presents empirical results suggesting that higher inflation is associated with lower real dividends and lower required real equity returns in the future. Therefore, positive shock to CPI leads to a reduction in the assets returns, that is, a contemporary reduction of the assets prices and a reduction in the future cash flows. These reactions would be consequence of the worsening macroeconomic scenario predicted by positive shocks in CPI and yield curve.

4.5
Final Discussions

The most influential expansions of the CAPM are the Fama and French three- and five-factor models. They propose new factors that once added to the model with the excess market returns, would help explaining the cross-section of portfolio returns. The first one, proposed in 1993, started a broad debate on the economic meaning of the proposed factors. FF identify some common risk factors in stocks returns in addition to excess market return, namely the effects of size (SMB) and value (HML). Later on, in 2015, they expand this set of factors including the effects of operational profitability (RMW) and investment (CMA).

FF (1996) suggests that we could interpret their factors as proxies for state variables, whose innovations describe the future investment opportunity set. Therefore, these innovations should command a risk premium and should be correlated with Fama-French factors. In other words, if betas with respect to innovations in a convenient set of state variables are important determinants of average returns, then there should be a statistically significant price of risk associated with those loadings.

This paper makes three contributions to the literature. First, we use a previous result in the literature, which shows that shocks to the aggregate dividend yield and term spread, default spread, and one-month Treasury-Bill rate are proxies for HML and SMB factors. We show that this result holds, no matter what the sequence of the variables at the factor vector is for the VAR that estimates the innovations of those state variables. As a second result, we show that the above result does not hold for the five-factor model, specifically, the above set of state variables do not proxy for RMW, the operational profitability factor. To overcome this situation we propose to add innovations in CPI to the set of state variables. We show that this new set of state variables proxy for SMB, HML, RMW and CMA. Finally, our third result shows that the set of state variables can be described only by innovations in CPI and the slope of yield curve, once there is a relevant correlation among some of these five variables. We show empirical evidence that portfolios returns are significantly correlated with unanticipated shocks in CPI and the slope of yield curve. This is an important result as these two variables are the main indicators of future changes in monetary policy. These results also suggest that an asset-pricing model in which the factors are the excess market return, the unexpected shocks to CPI and the yield curve slope, captures common time-varying behaviors in portfolio returns better than both the Fama and French (2015) and Petkova (2006) models.

5
Conclusion

This Thesis elaborates on important issues about portfolios returns and a specific modeling methodology proposed by Fama and French in 1993. It studies the possibility that the innovations in the average market variance, decomposed into two factors, one representing the average market variation and another representing the average market correlation, could increase the explanatory capacity of the three-factor model with respect to the excess returns of brazilian stock portfolios. It also studies the ability of the five-factor model to best explain stock portfolio returns in emerging market economic blocks relative to the original CAPM and the three-factor model. Finally, the study shows that innovations in the inflation index and innovations in the slope of the interest curve are proxies for size, value, profitability, and investment factors, and, together with excess market returns, explains cross-section of excess returns on stock portfolios better than the five-factor model.

The first subject studied propose that the idiosyncratic volatility (IV) of an asset can be seen as a factor of risk and could command a risk premium. Following, therefore, the intuition of Ang et al. (2006) that market aggregate volatility is priced, even though it exhibits contradictory behavior, and of Chen and Petkova (2012), who, in order to explain this contradiction, propose to break market aggregate volatility up into average variance and average correlation, it was analyzed whether idiosyncratic volatility is priced in the Brazilian market.

It occurs that, for US data, the risk premium commanded by average variance is significant and negative. The main explanation indicated by Chen and Petkova (2012) for a negative premium is the high level of investment in research and development by companies with a high level of IV. Portfolios composed of these companies would act as a hedge against deterioration of the environment and, thus, would have lower returns expectations. As the volume of investment of research and development recorded in Brazil is significantly reduced, if compared with that recorded in the United States, the expected result was that the Brazilian premium was positive. In fact, this occurs, and the risk premium commanded by exposure to average variance, according to the results found, is statistically significant and positive.

The second subject studied came from the growing integration of world stock markets in the last decades, which increases the importance of emerging markets as a field of research. Therefore, this study is an effort to reveal more evidence on emerging markets using data from 12 countries divided into Latin America, Asia, and East Europe economic blocks. We present results for how size, value, profitability, and investment affects the excess returns on stock portfolios after the 2008 global economic crisis. Basically, this work compares the three-, four- and five-factor models proposed by Fama and French (1993, 2015) and find which one better fits data. It finds clear evidence that the increase in the RHS of the equations from the CAPM and the FF models, improves the fit of the models. The study shows that the fit of the four- and five-factor models are very close in almost all cases, which seems to be evidence of a redundant effect between HML (high minus low) and RMW (robust minus weak) and CMA (conservative minus aggressive). This result is similar to those in Fama and French (2015). The evidence suggests that a four-factor model that drops the HML performs as well as the five-factors model, or slightly better. As a third result, it presents a clear size effect on excess stock market returns, and few evidence of the other effects. We do not find that value and profitability have clear effects and find some slight effect of investment. These results are not consistent with those reported for developed markets. However, they are not surprising either, considering all of the economic and political problems that most of these countries face and the low number of liquid stocks in most of its markets, which makes for a suboptimal diversification in its portfolios. Finally, as a fourth contribution, it finds that local factors perform better than US and global factors in explaining the returns of diversified portfolios, showing evidence of emerging market segmentation.

Our results are important because they show that emerging economy investors are pricing assets, to a certain extent, with a similar rationality as investors in developed economies do. This means that these markets are getting closer to developed markets in the sense that other disturbances, such as political and social issues, seem to be growing less relevant to investors than financial indicators are. It is one signal of the growing maturity of their economies and act as an incentive for international investments.

The last subject is based on Fama and French (1996) suggestion that one could interpret their factors as proxies for state variables, whose innovations describe the future and should be correlated with Fama-French factors. In other words, if betas with respect to innovations in a convenient set of state variables are important determinants of average returns, then there should be a statistically significant price of risk associated with those loadings.

Making an approach out of this concept, the study presents three interesting findings. First, it uses a previous result in the literature that presents a set of state variables that are proxies for SMB and HML factors, and shows that this result holds, no matter which the sequence of the variables at the factor vector is for the VAR that estimates the innovations of those state variables. As a second result, it shows that the above result do not hold for the five-factor model, specifically, the above set of state variables do not proxy for RMW, the operational profitability factor. To overcome this situation is proposed to add innovations in CPI to the set of state variables. The results show that this new set of state variables proxy for SMB, HML, RMW and CMA. Finally, its third result shows that portfolios returns are significantly correlated with unanticipated shocks in CPI and the slope of yield curve. This is an important result once these two variables are the main indicators of future changes in Monetary Policy. An asset-pricing model in which the factors are the excess market return and the unexpected shocks to CPI and the yield curve slope captures common time-varying behaviors in excess portfolio returns better than Fama and French five-factors model (Fama and French, 2015).

6
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