Pontifícia Universidade Católica do Rio de Janeiro

Leandro de Miranda Gomes

Term Structure of Variance and Dividend Returns

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Economia of the Departamento de Economia , PUC–Rio as a partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Ruy Ribeiro

Rio de Janeiro
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Abstract

Gomes, Leandro de Miranda; Ribeiro, Ruy(advisor). **Term Structure of Variance and Dividend Returns**. Rio de Janeiro, 2016. 32p. MSc. Dissertation — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

We extend the literature on the term structure of variance risk price and dividends strips. First we show that a substantial amount of S&P's equity premium is on the medium and long run dividends, contrary to previous literature. Then we indicate that market aggregated liquidity is relevant for the returns of variance related assets. We also provide some insights on the connection between these assets and show that both the slope and level of the variance risk curve are good predictors for monthly returns. Finally, we show that a properly calibrated version of Drechsler and Yaron (2011) can account for most of these stylized facts, as mean returns, Sharpe ratios and returns correlations.

Keywords

Risk Premium; Variance Risk; Dividend; Long run risk; Liquidity;

Resumo

Gomes, Leandro de Miranda; Ribeiro, Ruy(orientador). **Estrutura a Termo dos Retornos de Variância e Dividendos**. Rio de Janeiro, 2016. 32p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Nós adicionamos à literatura da estrutura a termo dos preços de risco de variância e parcelas de dividendos. Primeiro mostramos que uma parcela substancial do prêmio do S&P está nos dividendos de médio e longo prazo, contrário a literatura anterior. Depois, indicamos que a liquidez agregada do mercado é relevante para o retorno dos ativos de variância. Também provemos evidências da conexão destes diferentes tipos de ativos, ao mostrar que a a inclinação da curva de preços de variância é um bom previsor para retornos mensais. Por fim, mostramos que uma versão do modelo de Dreschler e Yaron (2011) é compatível com os fatos estilizados que encontramos, além de ser possível de capturar os retornos médios, Sharpe Ratios e correlações dos retornos.

Palavras–chave

Prêmio de risco; Risco de Variância; Dividendos; Risco de Longo Prazo; Liquidez;

Contents

List of Figures

List of Tables

1
Introduction

Since Mehra and Prescott (1985) introduced the equity premium puzzle and the predictability of returns literature originated, finance researchers have been trying to understand what kind of underlying theoretical explanation could realistically explain enough time-varying risk-premium in order to match empirical‘s apparent excessive stock returns. Moreover, different explanation were provided, as Cochrane and Campbell‘s (1999) habit-formation utility, Bansal and Yaron’s (JF, 2004) Long Run Risk model and Rare Disaster Risks model by Barro (QJE, 2006) and more recently Wachter (JF, 2013) and Gabaix (QJE, 2012). Each of the proposed solutions would model the time-varying risk premium through different channels, like a time-varying risk probability or a time-varying risk aversion, but there is not so far a consensual answer for the puzzle.

Even so, only recently the term structure of agent‘s expectations over the equity market has received literature’s attention, thanks to new proprietary derivative’s databases that can be used to infer prices for both dividends and variance over the time. Not only this nascent literature can shed light on which time-horizon is the risk premium, but it can also be used to check aforementioned model‘s consistency, both quantitatively and qualitatively.

For example, through options implied prices, Binsbergen, Brandt and Koijen (AER, 2012), BBK, calculated market’s prices for dividend strips, i.e. the price today of each of future’s dividends, and found that the equity premium is concentrated on the short term, which is inconsistent with many of proposed solutions. On the other hand, Dew-Becker, Giglio, Le and Rodriguez (2015), DGLR, provide evidence that the market only hedge shocks to realized equity market’s variance, while hedging against long term expected variance shocks are free, which also contradicts most of prevailing models on finance and some new macroeconomic models, where shocks to uncertainty are important drivers of the business cycle.

Using a different and more conclusive instrument to calculate dividend strips, we show that the S&P’s equity premium is actually on the long term or at best homogeneously spread through the curve, against previous results. On top of that, we also confirm DGLR result of higher prices for hedges against realized variance, using a similar but longer database both in terms of maturities available and time-series size. However, our results show that the argument for free-hedging against expected variance shocks is weak.

Moreover, at least to our knowledge, we are the first paper to use both the term structures of dividends and variance returns together, which can shed some light on the properties of the risk premium. We highlight that the short term variance risk is considerably more important for the whole term structure of equity premium than longer maturities. On the same line, we also show that both the slope and slope of the term structure of variance prices predict dividend's returns, which can be interpreted as a component of the time-varying probability of crisis embedded on these variables.

We also acknowledge that even though the prices of our derivatives should be highly influenced by liquidity risk, the lack of data on volume and bid-ask prices restricts our ability to circumvent this source of risk when analyzing the data. Nevertheless, we use Pastor and Stambaugh's (2003) traded liquidity factor to show that at least when concerned to market's aggregated liquidity, basically only the one month variance return is affected. This, together with the above results, indicate that the short-term variance price is a source of two components of risk-premium: one related to purely variance crisis risk, and other related to the aggregated liquidity risk.

It is worth emphasizing that one of the main contributions of this paper is to extend the reach of Bansal and Yaron (2004) long run risk models, which basically rely on Epstein-Zin utility functions and exogenously consumption and dividend growth dynamics that are subject to small but very persistent Gaussian shocks that generate the time-varying risk premia while maintaining low risk-free rates. This kind of model was already used to price other assets, like US Treasury bonds on Piazessi and Schneider (2006), exchange rate volatility on Colacito and Croce (2011), and the Variance Premium, i.e. the difference between one month priced and realized volatility as on Dreschler and Yaron (2011).

Likewise, we show that a long run risk model with jumps, very similar to Dreschler and Yaron (RFS, 2011), DY, albeit differently calibrated, can quantitatively match the term structure of equity‘s premium, maintaining higher relative importance for short term variance risk against longer-maturities, while maintaining the original results. Also, some results are improved when corrected by the liquidity risk, which seems reasonable as the construction does not model this kind of dynamics. This is needed, for example, to show that DY's model is also capable of matching the correlation matrix of dividends and variance returns, which adds even more robustness to model's ability to explain asset prices.

The remainder of this is organized as follows. Section 2 discuss the term-structure of the Equity Premium. Section 3 focuses on the sources of the risk premium, showing how the term structure of variance prices can help explain the equity premium. Finally, on the last section we show the ability of DY's model to match data's moments.

2
Term Structure of the Equity Premium

The price of an stock in time t, P_t, is the expected sum of all future dividends discounted by each correspondent stochastic factor, M_j. We call each parcels of the sums of a dividend strips.

$$P_t = E_t \left[\sum_{i=1}^{\infty} M_{t+i} D_{t+i} \right] \tag{2-1}$$

Even though asset pricing literature has been trying to provide a convincing answer for the reason equities returns are historically higher than conventional finance models would imply, in spite of limited data availability, little attention has been given to which time-horizon is important for prices formation. Therefore, in the same line of thought, so far we have scarce information on if shocks that alter the long-term marginal investor‘s views of expected discounted dividends are important, for example, or only short-term related news are relevant. What we try in this section is to shed light o this subject, which can be of the interest of finance practitioners, as well as *policy makers*, specially in today‘s era where Central Banks actively affect asset prices through forward guidance on long-term interest rates.

As far as we know, the first work to formally address this question is the aforementioned BBK's paper, which uses options data to infer the implied short term, D_{ST}, and long term dividends, D_{LT}, prices. Formally, the objects they are aiming to construct are the following , where i corresponds to month units:

$$D_{ST} = E_t \left[\sum_{i=2}^{24} M_{t+i} D_{t+i} \right] and\ D_{LT} = E_t \left[\sum_{i=25}^{\infty} M_{t+i} D_{t+i} \right] \tag{2-2}$$

In other words, their dataset allows them to form two blocks of dividends strips prices, the short term being the implied by options prices of the sum of the next 2 years of monthly dividends, and the long-term is the residual prices needed to make (1) hold. They find that from January 1996 to October 2009, a substantial amount of the S&P‘s equity premium is on the short term, which dividend‘s strip generated an annualized 10% return, against only 3.2% from the index. Therefore, they argue, long-term returns would have had to be lower to make the identity (1) hold.

What we show next, using a different kind of dataset, is that if we can disentangle BBK‘s residual long-term dividend strips returns into annual ones of maturities of 1-10 years, results can be interpreted differently.

2.1 Dividend Swaps

In this section we show how we construct the following Dividend Strips and compare the results to BBK‘s.

$$D_{12} = E_t\left[\sum_{i=1}^{12} M_{t+i}D_{t+i}\right], D_{24} = E_t\left[\sum_{i=13}^{24} M_{t+i}D_{t+i}\right], \qquad (2\text{-}3)$$
$$D_{36} = E_t\left[\sum_{i=25}^{36} M_{t+i}D_{t+i}\right], ..., D_{108} = E_t\left[\sum_{i=97}^{108} M_{t+i}D_{t+i}\right]$$

Our strategy to calculate dividend strips prices has some advantages over BBK‘s, albeit with some shortcomings. Specifically, using Dividend Swaps, which will be formally described below, not only we can have more granularity on the "long-term", but we can also see the exactly market‘s price of a future Dividend. However, being a forward contract, what we get is not exactly a dividend strip. Moreover, in order to calculate the price of a dividend swap today, we proceed by simple discounting it‘s price by the equivalent maturity US Treasury Bill yield.

Formally, Dividend Swaps (DS) are financial derivatives where we exchange realized dividends RD of a specific period, i.e. year 2020, for a strike price DS. Therefore, the payoff of implementing a strategy of a DS is simply:

$$Payoff_t^n = \sum_{j=t+n-1}^{t+n} RD_j - DS_t^n$$

Usual maturities traded on financial market‘s for the main Equity Indexes range from 1 to 9 years of future‘s dividends. Here, we use a proprietary database from an important derivative’s player for US S&P500 Index Dividend Swaps including these most common maturities mentioned. Our data ranges from January 2003 to June 2014.

Because we have data of yearly maturities, i.e. between 12 and 24 or 48 and 60 months ahead, in order to calculate monthly returns, we follow the literature and interpolate our maturity-wise yearly dividend swaps into monthly ones. In other words, we now have the prices for dividends between months 12 and 24 as well as 13 and 25 and therefore after. Finally, the monthly return for holding an n Dividend Swap is:

$$Return_{t+1}^n = \frac{DS_{t+1}^{n-1} - DS_t^n}{DS_t^n}$$

When we take a look at the time series of dividend swaps in Figure 1, the longer the maturity, the higher the expected dividend, and with the exception to the Great Financial Crisis, there was always an implied dividend growth priced on the market.

Figure 2.1: Dividend Swap Time Series

Notes: We present the 1, 3,5,7 and 9 years Dividend Swap time series from January 2003 to June 2014. Dividend Swaps are financial derivatives that allow us to buy future dividends of different maturities, therefore, allowing us to construct it's term structure. Data is proprietary from a important derivatives player.

Finally, on table 1, we show the main result of the session, which is the fact that in our dataset, the equity premium is concentrated on the medium and long-term parcels of the curve. While the excess annualized return of S&P 500 is of 6.9 %, the mean monthly returns of the dividend strip of 12 and 24 months is only of 5%. Nonetheless, all other dividend strips of longer maturities average more than 7% during this time span. Therefore, from our perspective, it is the medium and long-term that drives equity premium so high, in contrast to previous results. Also, it is true that our results may be sensible to the term structure of treasury yields, if the stochastic factor of discount is not completely independent of the risk-free assets. Indeed, there is a difference between dividend swaps and strips returns, as would be reasonable to expect. However, the upward-sloping characteristic of both the dividend swaps returns curve indicate that at least qualitatively, our results should hold. In other words, it seems reasonable to expect that at least the long-part of the curve absorbs a non-trivial parcel of the equity's premium.

However, it should be taken notice that this effec can be attributed to two different variables. The first is derived from the cash-flow, in this case the expected future's dividend, and the second from the correspondingly stochastic discount factor. We cannot know for sure which one of the two is the main

Table 2.1: Term Structure of Equity Premium

		Rm-rf	1y	2y	3y	4y	5y	6y	7y	8y	9y
Dividend Swaps	Mean Return	6.9%	4.4%	5.9%	6.9%	7.4%	8.1%	9.1%	9.6%	9.8%	10.3%
	Annualized Sharpe Ratio	0.48	0.47	0.47	0.5	0.49	0.49	0.52	0.52	0.51	0.51
Dividend Strips	Mean Excess Return	6.9%	5.0%	7.0%	8.7%	9.9%	10.9%	12.2%	12.9%	13.4%	13.9%
	Annualized Sharpe Ratio	0.48	0.56	0.58	0.67	0.68	0.72	0.76	0.76	0.77	0.78

Notes: Here we present the mean returns of Dividend Swaps and Dividend Strips from January 2003 to June 2014. Dividend Swaps are financial derivatives that allow us to buy future dividends of different maturities, therefore, allowing us to construct it‘s term structure. In order to construct Dividend Strips, i.e., the price of today‘s Dividend Swaps, we use the US Treasury Bills implied interests rate for each of the maturities. Proprietary‘s data is from an important derivatives player.

driver of this result. Perhaps, people were expecting, in physical measure, a great increase in the dividends in the future, which would drive by itself the price of future‘s dividends higher. On the other hand, this may only indicate that people demand a higher return to hold a future‘s dividend because all the associated risks with this asset.

Unfortunately, looking only at dividend swaps is not enough to answer this question. Thereafter, we now look at the term structure of variance swaps, which are very similar to dividend swaps contracts, but instead of betting on future‘s dividend, we are now concerned on the equity market‘s variance, which should be more correlated to stochastic factor of discount.

3
Sources of Risk Premium

On economic models, some kind of variance is usually a source of risk premium. On CAPM, market‘s variance is related to the stochastic discount factor, for example. In more recent models, as the one we are going to introduce on the next section, market‘s variance is related to probability of a sizable jump on consumption and dividend‘s persistent trajectory. Here we try to assess if the term structure of variance risk prices is related to the term-structure of the equity premium, in order to try to separate the two channels that can be driving the extra medium/long term equity premium.

3.1
Variance Swaps and Zero-Coupons Variance Swaps

Variance Swap is a financial instrument where we exchange the realized variance RV over n periods for a strike price VS^n with the payoff defined as:

$$Payoff_t^n = \sum_{j=t}^{t+n} RV_j - VS_t^n$$

Usual maturities are 1, 2, 3, 6, 12 months as well as 1-12 years. Under regular conditions, the fair price of a maturity n Variance Swap is

$$VS_t^n = \mathbb{E}_t^Q \left[\sum_{j=1}^{n} RV_{t+j} \right]$$

Where Q indicates the risk neutral probability measure.

Following Dew-Becker, Giglio, Le and Rodriguez (2015), using a non-arbitrage argument, we construct the instrument for expected variance n periods ahead, which we call the Zero-Coupon Variance Swap, as:

$$Z_t^n = \mathbb{E}_t^Q [RV_{t+n}] = VS_t^n - VS_t^{n-1}$$

Also $Z_t^0 = RV_t$, the realized variance and $Z_t^1 = VS_t^1$, simply the price of the 1-month variance.

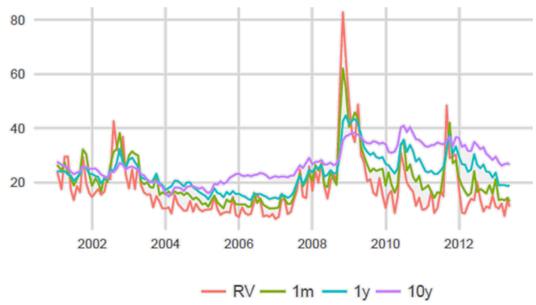
The return of betting on the price of expected variance in t for n periods ahead, which we expect to be negative on average, is then simply

$$Return_{t+1}^n = \frac{Z_{t+1}^{n-1} - Z_t^n}{Z_t^n}$$

We use a proprietary database on Variance Swap with maturities of 1 month, and 1 to 10 years, from the same important derivatives player of dividend swap. Our time-series range from January 2001 to May 2013. We use the same interpolation to achieve the monthly returns as in the Dividend Swaps.

The main difference of our database from DGLR is that they have shorter and fewer maturities after 1-year contracts. For long-term variance swaps contracts, for example, their database covers only 2008-2013, which may be a small window due to the financial crisis extreme event, as we can see on the Figure 2.

Figure 3.1: Variance Swap Time Series



Notes: In the Figure, we present the Realized Volatility (RV), 1-month annualized volatility and 1 and 10 years annualized volatility from the Variance Swap. Variance Swaps are financial derivatives that allow us to buy future realized variance of different maturities, therefore, allowing us to construct it's term structure. Time Series is from January 2001 to May 2013. Variance Swaps data is from a proprietary important derivatives player.

Consequently, our database is more suitably to cover medium and long term assets. Nonetheless, our result resembles DGLR average term structure of prices, an expectedly upward slope, which intuitively may be due to a higher probability of extremes events as time go by, as seen in Figure 3. On the other hand, realized variance shocks lead to a downward slope curve, because the probabilities of the effects of this extreme event fade away with time, as happened in the epicenter of GFC, and seen in Figure 4.

In the Figure, we present the average prices for annualized volatility from the Variance Swap databases for the Realized Variance, 1-month Variance Swap, and 1 to 10 years Variance Swap. Variance Swaps are financial derivatives that allow us to buy future realized variance of different maturities, therefore, allowing us to construct it's term structure. Time Series is from January 2001 to May 2013. Variance Swaps data is from a proprietary important derivatives player.

Figure 3.2: Average Variance Swap Prices

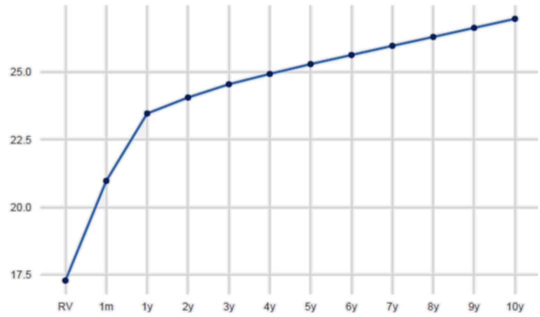
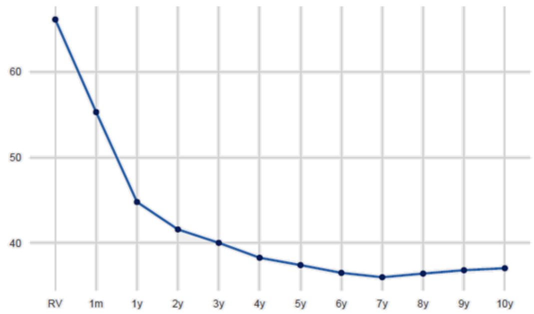


Figure 3.3: Variance Swap Curve at November 2008



Variance Swaps data is from a proprietary important derivatives player. In the Figure, we present the prices for annualized volatility from the Variance Swap databases for the Realized Variance, 1-month Variance Swap, and 1 to 10 years Variance Swap on November 2008. Variance Swaps are financial derivatives that allow us to buy future realized variance of different maturities, therefore, allowing us to construct it's term structure. Variance Swaps data is from a proprietary important derivatives player.

3.2 Variance Risk and the Equity Premium

With the proper introduction of Variance Swaps, we now turn back to the relation of the term structure of variance and dividends prices. The first evidence is that, as we can see on table 2, not surprisingly, the cross-correlations of dividend and variance swaps monthly returns are mostly negative, indicating that there may be some kind of priced variance risk, both on short and long-term dividends. In other words, people could be holding variance swaps to hedge against shocks to the future's dividend.

However, the magnitude of the 1-month variance swap return cross-correlation is significantly higher than other maturities, close to -40% against other's near to 0, which may be a signal that there are other than variance-related risks premiums associated with the stochastic discount factor. Nonetheless, the 1-month variance swap may be a more relevant source of

Table 3.1: Matrix of Zero-Coupon Variance Swaps and Dividend Swaps Cross-Correlations

DS/VS	1m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
1y	-42%	7%	9%	5%	5%	6%	2%	4%	4%	-1%	0%
2y	-39%	6%	3%	2%	-2%	0%	0%	0%	1%	-3%	-3%
3y	-40%	5%	1%	-1%	-4%	-2%	-1%	-3%	-1%	-4%	-5%
4y	-38%	3%	-2%	-3%	-7%	-4%	-5%	-7%	-6%	-8%	-8%
5y	-40%	0%	-5%	-6%	-9%	-7%	-8%	-9%	-7%	-10%	-10%
6y	-41%	1%	-4%	-6%	-9%	-7%	-10%	-10%	-6%	-10%	-9%
7y	-41%	2%	-4%	-6%	-11%	-9%	-11%	-12%	-9%	-12%	-11%
8y	-42%	4%	-2%	-4%	-10%	-8%	-9%	-11%	-8%	-13%	-13%
9y	-41%	4%	-2%	-4%	-9%	-8%	-9%	-10%	-7%	-13%	-14%

We show the cross-correlation between zero-coupon variance swaps and dividend-swaps for all available maturities. Variance and Dividend Swaps are financial derivatives that allow us to buy future realized variance and dividends of different maturities, therefore, allowing us to construct the it's term structure. Data is proprietary and from an important derivative's player.

risk for the term-structure of the equity premium, whatever the maturities of dividend swaps we consider.

This result also goes in line with DGLR results, where Sharpe Ratios for 1-month Variance Swap are significantly more negative than for longer maturities variance-swaps. We confirm that 1-month variance swaps appears, for some reason, to be significantly more risk-wise important for the whole term structure of equity premium. Therefore, the fact that Sharpe Ratios for longer maturities of variance swap returns are close to zero should not be come as a surprise.

While we confirm DGLR results with our longer and richer in maturities database, we would only like to emphasize that the close to zero long variance swaps Sharpe Ratios should not be interpreted as unimportant sources of risks for the equity premium. Nor they should be seen, as in DGLR, as free-hedges against shocks to expected variance, because the impact of the Great Financial Crisis should not be overlooked. Taking away this extreme and unique event, and all Sharpe ratios would be positive again.

3.3 Why Variance Risk?

One of the possible hypothesis for why the term-structure of variance risk should be important for the equity premium is that there may be some information embedded on its term structure about the conditional probability of a jump or crisis on the market or the real economy. Here we follow the same line of reasoning of DY which found that Variance Premium (VP) predicts

Table 3.2: Predictability of Dividend Swaps

	1y	2y	3y	4y	5y	6y	7y	8y	9y	Rm-Rf
β_1	-26.52	-37.10	-38.71	-38.04	-37.88	-36.57	-36.43	-35.20	-31.03	-47.12
t-stat	-2.61	-3.28	-2.42	-1.62	-1.44	-1.19	-1.08	-0.89	-0.61	-4.42
β_2	14.73	23.78	28.22	32.15	40.90	50.15	56.55	60.11	65.30	29.79
t-stat	1.67	2.57	2.39	2.30	2.64	3.02	3.28	2.79	2.35	3.80
R2	0.27	0.30	0.24	0.21	0.21	0.20	0.20	0.18	0.16	0.49

We present evidence on the predictability of the long-term slope of the Zero-Coupons Variance Swaps Prices on 2 years accumulated returns for all available Dividend Swaps, as well as market‘ts excess return. In order to do that, we regress the dividend swaps returns for all available maturities on the level (first variable), the 1-month zero-coupon variance swap, and the slope (second variable), the 9 years Zero-Coupon Variance Swap minus the 1 year. All our t-stats are corrected by Newey-West.

aggregated market returns, linking the VP to market's perception of the probability of a jump on consumption and dividends growth, which in turn creates a conditional equity's market risk premium.

We use both the slope of the variance curve, measured by the 9 year Zero-Coupon Variance Swap price minus the 1 year, and the level of the curve, here the 1 month variance swap price, because our results above indicate that there is some independent dynamics on the very short-term part of the curve. We use the 2 years accumulated returns on each of the available dividend swaps, as well as on the market. All our t-stats are corrected by Newey-West.

What table 3 indicated is that both slope and level of the curve indeed have predictability power over the 2 years returns. Also, even though statistically and economically relevant for the short term of the dividend swaps returns, the 1-month variance swap prices have limited impact on the long part of the curve, also in line with our previous results. On the other hand, the slope of the curve, is a good predictor for the whole curve, including market's.

3.4 Variance or liquidity risk?

One question that naturally arise is how important the fact that some of these assets are not highly liquid is on the prices and returns. One could argue, for example, that a substantial amount of the risk of these assets is from the fact that because their liquidity is low, there should be an extra compensation for holding them, generating an extra premium on top of more "fundamental" sources of risk. Indeed, there is anecdotal evidence that the less liquid instruments are the long-term ones, both variance and dividend wise. However, as the databases are proprietary, we do not have access to volumes traded and bid-ask spread that could help us answer to what degree there is an relevant extra compensation for the lack of liquidity.

Nonetheless, even though we can not analyze the impact of each of liquidity‘s problems specifically for each of the assets, we can at least understand their sensitivity to aggregated market liquidity. In other words,

Table 3.3: Liquidity effect on Variance Swaps returns

	Alpha	Market	SMB	HML	TL - P
VS 1M	-0.14	-0.12	0.03	0.04	-3.80
	0.03	0.00	0.29	0.16	0.02
VS 1Y	0.00	0.00	-0.01	0.01	0.30
	0.84	0.83	0.31	0.06	0.38
VS 2Y	0.00	0.00	0.00	0.01	0.16
	0.97	0.90	0.60	0.05	0.56
VS 3Y	0.00	0.00	0.00	0.01	0.19
	0.99	0.88	0.45	0.02	0.44
VS 4Y	0.00	0.00	0.00	0.01	0.13
	0.85	0.57	0.53	0.08	0.58
VS 5Y	0.00	0.00	0.00	0.01	0.20
	0.89	0.59	0.42	0.06	0.41
VS 6Y	0.00	0.00	-0.01	0.01	0.17
	0.84	0.24	0.35	0.17	0.54
VS 7Y	0.00	0.00	0.00	0.01	0.18
	0.95	0.21	0.71	0.17	0.46
VS 8Y	0.00	0.00	0.00	0.01	0.30
	0.95	0.47	0.75	0.14	0.24
VS 9Y	0.00	0.00	0.00	0.01	0.27
	0.99	0.09	0.92	0.05	0.29
VS 10y	0.00	-0.01	0.00	0.01	0.33
	0.72	0.02	0.86	0.01	0.22

Here we regress the monthly returns of zero-coupons variance swaps on the 3 Fama-French factors plus the market aggregated market liquidity traded factor by Pastor and Stambaugh's(2003). The first line of each column constitutes β and below it is the respective p-value.

does low aggregated-market liquidity generates high sensitivity on variance and dividend swaps returns? In order to try to answer this question we use Pastor and Stambaugh's (2003) (TL - P) traded aggregated liquidity factor to assess its impact on our database.

Perhaps surprisingly, when regressing our monthly returns on Fama French 3 factors plus TL - P, only the one month variance swap and 13 months dividends swap returns are significantly affected by market's liquidity, as seen on the table 4 and 5. This does not mean that other returns are not influenced by their own illiquidity; however the results indicate that the relevance of market's aggregated liquidity is concentrated on the short-term curve of the variance price.

Economically, the non-trivial result is even thinner: only the one-month variance swap returns gets sensible changes after subtracting the liquidity factor parcel. Intuitively, this may be due to the fact that lower market liquidity can generate greater realized variance by construction, which would

Table 3.4: Liquidity effect on Dividend Swaps returns

	Alpha	Market	SMB	HML	TL - P
DV 1y	0.00	0.00	0.00	0.00	0.16
	0.88	0.00	0.79	0.76	0.00
DV 2y	0.00	0.01	0.00	0.00	0.08
	0.75	0.00	0.76	0.35	0.26
DV 3y	0.00	0.01	0.00	0.00	0.13
	0.69	0.00	0.84	0.26	0.09
DV 4y	0.00	0.01	0.00	0.00	0.12
	0.69	0.00	0.77	0.22	0.18
DV 5y	0.00	0.01	0.00	0.00	0.08
	0.65	0.00	0.73	0.31	0.35
DV 6y	0.00	0.01	0.00	0.00	0.08
	0.59	0.00	0.66	0.27	0.41
DV 7y	0.00	0.01	0.00	0.00	0.09
	0.62	0.00	0.58	0.26	0.36
DV 8y	0.00	0.01	0.00	0.00	0.12
	0.67	0.00	0.60	0.32	0.24
DV 9y	0.00	0.01	0.00	0.00	0.11
	0.64	0.00	0.49	0.35	0.30

Here we regress the monthly returns of dividend swaps on the 3 Fama-French factors plus the market aggregated market liquidity traded factor by Pastor and Stambaugh‘s(2003). The first line of each column constitutes β and below its respective p-value.

consequently influence the returns of 1-month variance swaps. Taking the great financial crisis as an example, there was a relevant lack of liquidity on the market that mechanically could have increased the realized variance.

Accordingly, a substantial amount of the 1-month variance risk is related to the aggregated market liquidity. When we subtract the aggregated market liquidity effect of the 1-month variance swaps and re-calculate new correlations between this "new" instrument return and the dividend swaps, results actually become much smaller, as seen in table 6. On the same line, the new Sharpe Ratio of this aggregated market liquidity free 1-month variance swap is lower, at -1.53. Furthermore, the need to include the 1-month variance swap on the slope of the term-structure of variance risk indicate that this source of risk is important to generate not only a time-varying probability of a jump, but also a time-varying probability of a liquidity squeeze.

Therefore, we conclude that the 1-month variance swap is a source of two potential risks: pure realized variance and aggregated-market liquidity.

Table 3.5: Cross-correlation of 1-month variance swaps returns free of the aggregated market liquidity and the dividend swaps

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1-month VS*	-0.08	0.06	-0.06	-0.05	-0.06	-0.08	-0.08	-0.08	-0.08

Here we subtract the impact of the market aggregated liquidity on the 1-month variance swap return using the coefficient of the regression above.

4
Long-run Risk with Jumps model

4.1
Framework

A common feature on this nascent literature of term-structure of dividends and variance prices is to test if standard finance models can provide the same kind of stylized facts seen on data. For instance, DLGR show that Dreschler and Yaron (2011) and Wachter‘s (2013) models fail to generate close to zero Sharpe Ratios of Variance Swaps of longer maturities, while Gabaix's (2012) model of rare disaster can match this stylized fact, even though failing in quantitatively match other moments. On the other hand, BBK show that Campbell and Cochrane (1999), Bansal and Yaron (2004) and Gabaix (2009) generate too much equity premium on the medium and long term, which would be at odds with their data, while not with ours. Thereafter, both these papers are very skeptical on the ability of standard financial models to provide adequate answers to the term structure of risk premium. Our results, however, are clearly more positive.

Here we focus on DY model, which is an extension of Bansal and Yaron (2004) long-run risk mode, where agents have preference for early uncertainty and thereafter are willing to pay to protect themselves against news shocks to the state variables which are governed by a persistent long-term risk. DY new feature is to make this long-term risk vulnerable not only to Gaussian, but also, Jump shocks. They demonstrate, then, that the time-variation on the economic uncertainty generate positive variance premium that can predict excess stock market returns. Also, this is considered a very successful model for being able to quantitatively match mean, volatility, skewness and kurtosis of consumption growth and stock market returns, as well as mean and standard deviation of Variance Premium.

We show that if we properly calibrate DY model to match the term-structure of Variance Swaps, instead of focusing on the 1-month Variance Premium as in the original paper, while former results are maintained, now we can also match variance and dividend swaps returns. Therefore, not only we have an upward sloping equity premium up to 9 years, and a closer to zero relevance of long-term variance risk,contraty to what DGLR found, but we can also quantitatively match the new Sharpe Ratios for variance and dividend swaps and Variance-risk Prices.

4.2 The Model

First, the model starts with a maximization problem of a representative agent with Epstein-Zin utility, V, subject to a budget restriction of an endowment W. He can either consume this endowment or reinvest a portion for a total return of $R_{c,t+1}$

$$\begin{aligned} &\max && V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \\ &\text{subject to} && W_{t+1} = (W_t - C_t)R_{c,t+1} \end{aligned}$$

The dynamics of the economy is as follow: both consumption (Δc) and dividend (Δd)) growth are exogenously modeled with a common long-run risk variable (x) and subject to non-independent Gaussian-Shocks. The Long-run risk is a very persistent variable subject to both Gaussian and jump shocks, which generates large and infrequent negative pulses and more often small positive effects.

$$\Delta c_{t+1} = \mu_c + x_t + z_{c,t+1}$$

$$x_{t+1} = \rho_x x_t + z_{x,t+1} + J_{x,t+1}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + z_{d,t+1}$$

The other main component of the model is the σ^2 , with a long-run mean $\bar{\sigma}^2$, which is also a persistent process that determinate the intensity and probability of jumps and Gaussian shocks on the long run risk variable.

$$\bar{\sigma}^2_{t+1} = \rho_{\bar{\sigma}}\bar{\sigma}^2_t + z_{\bar{\sigma},t+1}$$

$$\sigma^2_{t+1} = \rho_\sigma \sigma^2_t + (1-\rho_{\bar{\sigma}})\bar{\sigma}^2_t + z_{\sigma,t+1} + J_{\sigma,t+1}$$

The Gaussian shocks, z, are calibrated so that there are correlations only between consumption and dividend growth processes, but their intensity is time-varying and depend on the σ^2.

$$z_{t+1} = (z_{c,t+1}, z_{x,t+1}, z_{\bar{\sigma},t+1}, z_{\sigma,t+1}, z_{d,t+1}) \sim N(0, h + H\sigma^2_t)$$

Specifically, the Jump Shocks,for each j, are compound-Poisson processes in the form below:

$$J_{i,t+1} = \sum_{j=1}^{N^i_{t+1}} \xi^j_i \qquad i = \sigma, x$$

Where ξ^j_i are conditionally independents. N^j_{t+1} is the Poisson counting process with intensity $\lambda_t = l_\sigma \sigma_t^2$ for both jumps. Therefore, we emphasize that σ_t^2 is the variable that determinate the expected number and also the intensity of all shocks, both jump or Gaussian, so that it will have a major role on creating the risk-premium trough its induced time-varying risk probability.

The calibration strategy was to follow Dreschler and Yaron closely. The key points are:

- Risk aversion coefficient: $\gamma = 9.5$
- Inter-temporal elasticity of substitution: $\psi = 2$
- $\xi^x \sim -exp(1/v_x) + v_x$, to achieve small and frequent positive but large and infrequent negative jumps
- $\xi^\sigma \sim exp(1/v_\sigma)$
- $l_\sigma = 0.75$ so that jump shocks arrive on average 0.75 times per years, and negative jump long run jumps once in 4 years
- Originally, Drecshler and Yaron calibrated the model to match 1-month Variance Swap ($l_\sigma = 0.8$) Here, I use the parameter to match long-term Variance Swap prices

What is the difference of our calibration and the original, which was also used on DLGR? While DY original results still hold, as seen in the next session, if we calibrate the average number of shocks to match the Variance Premium instead of the long-term price of variance, we end up overshooting the prices for the long-term variance swaps on the model, which is what DLGR found. However, because our intention is to match the long-term variance swaps, we calibrated the average number of shocks accordingly.

The results are based on 1000 simulations of 79 years of monthly observations. However, each variable is calculated using the equivalent sample size on the data.

4.3 Results

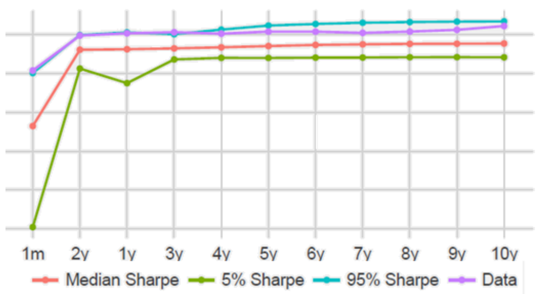
First, it is worth emphasizing that despite being slightly different calibrated, the original results are maintained as seen in table 7. Second, contrary to DGLR, we find that the model can properly match both variance Sharpe ratios as seen in figure 5. Therefore, the variance-risk is concentrated on the short term of the curve, just like on data. Moreover, Sharpe Ratios for medium to long-term maturities are close to 0, and positive results appear within the confidence interval, indicating that even though these risks are relevant and agents would like to hold hedges against them, states of nature where infrequent large jumps occur on the simulated economy are enough to generate in-sample positive returns, just like our data is affected by the Great Financial Crisis.

Table 4.1: Term-Structure of Variance-Risk Prices

	Data	Model		
Moments	Est	5%	50%	95%
$E[\Delta c]$	1.88	1.02	1.93	2.82
$E[\Delta d]$	1.54	-1.08	1.85	5.04
$sd(\Delta c)$	2.21	1.80	2.32	3.07
$sd(\Delta d)$	13.69	9.43	11.18	12.97
$ac1(\Delta c)$	0.43	0.21	0.44	0.64
$ac2(\Delta d)$	0.14	0.12	0.28	0.47
$cor(\Delta c, \Delta d)$	0.59	0.11	0.33	0.53
$E[r_f]$	0.82	0.51	0.92	1.50
$sd[r_f]$	1.89	1.04	1.47	2.84
$E[r_m]$	6.23	3.83	6.39	10.06
$sd(r_m)$	19.37	14.64	17.30	20.85
$sd(var_t(r_m))$	17.18	2.26	8.85	28.23
$ac1(var_t(r_m))$	0.81	0.65	0.80	0.91
$ac2(var_t(r_m))$	0.64	0.42	0.64	0.83
$E[VP]$	11.27	4.27	10.17	20.62
$sd[VP]$	7.61	3.16	9.74	26.61

Here we present the results for the term-structure of annualized zero-coupons variance risks. Results from the model are based on 1000 simulations of 78 years, but we take a sample of the same correspondingly real data. Confidence Intervals shown are 5-95%.

Figure 4.1: Variance Zero-Coupons Sharpe Ratios

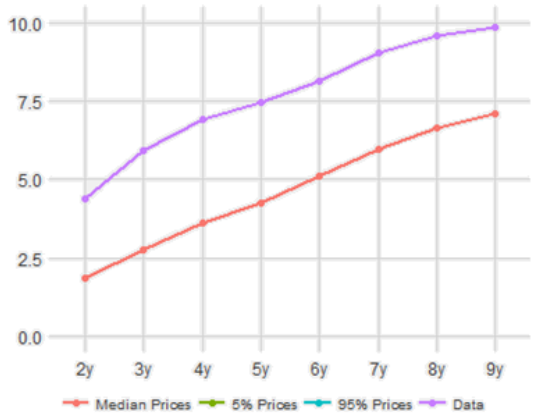


Here we present the same variables presented on DY's paper with our calibration. Results from the model are based on 1000 simulations of 78 years, but we take a sample of the same correspondingly real data. Confidence Intervals shown are 5-95%.

Also, even though the confidence intervals are considerable large, dividend swaps returns and Sharpe ratios are within them, as seen in figures 6. Nonetheless, the upward slope of the dividend swaps returns is at least a good qualitatively important feature of the model.

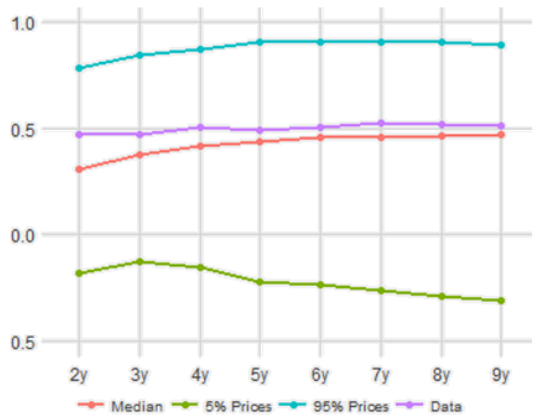
What about the results that appear to be sensitive to the liquidity risk on data? Because the model does not have any kind of liquidity-risk modeled, it is reasonable to expect that the model may fail in this respect. Indeed, the cross-correlations matrix is only matched, as seen in table 8, when we take make the 1-month variance swap correction for the aggregated market liquidity

Figure 4.2: Dividend-Swaps Mean Returns



Here we present the results for the term-structure of annualized mean returns of Dividend Swaps. Confidence Intervals of 5-95% are wide and does not appear on the plot for aesthetic reasons. Results from the model are based on 1000 simulations of 78 years, but we take a sample of the same correspondingly real data.

Figure 4.3: Dividend-Swaps Sharpe Ratios



Here we present the results for the term-structure of annualized Sharpe Ratios of Dividend Swaps. Results from the model are based on 1000 simulations of 78 years, but we take a sample of the same correspondingly real data. Confidence Intervals shown are 5-95%.

on the data. Here we show only the median correlations, which are basically close to zero, however negative correlations are within the confidence interval, even though the 1-month variance swap is never as high as in the non-liquidity corrected version of data. We suppress here the confidence intervals for these matrix for aesthetic reasons.

Finally, another interesting feature for the liquidity-free 1-month variance swap returns is that its new Sharpe Ratio on data is -1.53, much closer to the simulations median. Therefore, we have another good indicative of the existence of liquidity-risk that the model may not correctly capture if we do not provide the adequate correction.

The bottom line is that DY model framework does a very decent job

Table 4.2: Model's cross-correlations median results

DS/VS	1m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
1Y	-1.4%	-0.5%	-0.3%	-0.4%	-0.5%	-0.5%	-0.6%	-0.5%	-0.3%	-0.1%	-0.1%
2Y	-0.5%	-0.3%	0.1%	0.2%	0.0%	0.2%	-0.3%	-0.2%	-0.4%	-0.4%	-0.3%
3Y	-0.1%	0.2%	0.4%	0.6%	0.2%	-0.2%	-0.4%	-0.4%	-0.4%	-0.3%	-0.3%
4Y	0.7%	0.9%	0.9%	1.2%	0.9%	0.7%	0.2%	-0.2%	-0.5%	-0.4%	-0.4%
5Y	1.2%	1.1%	1.3%	1.0%	1.0%	0.6%	0.2%	0.1%	-0.2%	-0.2%	-0.3%
6Y	1.1%	0.9%	1.1%	1.2%	0.8%	0.7%	0.2%	0.1%	-0.1%	-0.3%	-0.4%
7Y	1.0%	0.4%	0.6%	0.7%	0.6%	0.3%	-0.2%	-0.3%	-0.3%	-0.6%	-0.7%
8Y	1.4%	0.8%	0.6%	1.0%	0.7%	0.8%	0.1%	-0.2%	-0.4%	-0.6%	-0.6%
9Y	0.7%	0.3%	0.3%	0.6%	0.5%	0.4%	0.0%	-0.3%	-0.4%	-0.4%	-0.4%

Here we present the median cross-correlations of the model's simulation, based on 1000 iterations of monthly 79 years each.

explaining not only the standard financial and economic data aforementioned, but also shows robustness to explain the term-structure of variance and dividends prices, albeit not being able to match non-corrected liquidity aspects, as we would expect.

5
Conclusion

This paper show that the equity premium is on the medium and long term, contrary to previous research. Also, regarding to variance expected and variance shocks, even though we confirm literature's results, we disagree on the interpretation that there free hedges against expected variance shocks. On top of that, the short-term variance risk premium is more important to the whole term-structure of all maturities dividends. This may be due to the fact that the slope of the term-structure of variance risk prices contain some kind of information of market‘s perception of the time-varying risk of a crisis. Also, we show that a portion of the explanations relies on the fact that this 1-month variance swap return is highly related the aggregated market‘s liquidity. Finally, another result of this paper is that a standard financial model, DY in this case, can match different kinds of market's and economics data both quantitatively and qualitatively, indicating that there is robustness to a long-run risk kind of explanation for the risk premium.

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