Pedro Bessone Tepedino

Protests, Concession and Repression in a Networked Society

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Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Economia of the Departamento de Economia , PUC–Rio as a partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Vinicius Nascimento Carrasco

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Prof. Vinicius Nascimento Carrasco
Advisor
Departamento de Economia — PUC–Rio

Prof. Leonardo Rezende
Departamento de Economia - PUC-Rio

Prof. Thierry Verdier
PSE - Paris-jourdan Sciences Economiques

Prof. Bernardo de Vasconcellos Guimarães
FGV-SP

Prof. Monica Herz
Coordinator of the Centro de Ciências Sociais – PUC–Rio

Rio de Janeiro, April 29^{th}, 2016

Pedro Bessone Tepedino

The author graduated in Economics from PUC-Rio in 2013.

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Abstract

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We develop a sequential game between groups of individuals taking part in a mass protest and a democratic government facing electoral constraints. The groups are connected by a network of participation externalities, as participation from individuals generate arbitrary heterogeneous externalities in members of other groups. This setting allows us to study a myriad of unexplored phenomena like how the presence of strong leaderships or radical groups affect protests' pattern of participation and the likelihood of repression. Our results explain in particular how the recent communication revolution affected protests' outcomes. In a nutshell, our results indicate that horizontal protests are more likely repressed and unpopular radical groups diminished the likelihood of ousting the incumbent from office, implying that the government will use any means at its disposal in order to consolidate radical groups.

Keywords

Protests; Social Networks; Political Economy; Repression;

Resumo

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Nós desenvolvemos um jogo sequencial entre grupos de indivíduos participando em um protesto de massa e um governo democrático enfrentando pressões eleitorais. Os grupos estão conectados por uma rede de externalidades de participação, uma vez que a participação dos indivíiduos gera externalidades heterogêneas arbitrarias em membros de outros grupos. Esta configuração nos permite estudar diversos fenômenos ainda não explorados, como o fato da presença de lideranças fortes ou grupos radicais afetam o padrão de participação popular nos protestos e a probabilidade de repressão. Nossos resultados explicam, em particular, como a recente revolução nas tecnologias de comunicação afetam os resutlados dos protestos. Resumidamente, nossos resultados mostram que protestos horizontais têm maior probabilidade de serem reprimidos e grupos impopulares aumentam a probabilidade de reeleição do incumbente, implicando que o governo usará quaisquer meios disponíveis para para consolidar grupos radicais.

Palavras–chave

Protestos; Redes Sociais; Economia politica; Repressao;

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1 Introduction

This paper studies how structural aspects of mass protests affect the likelihood of the government conceding, repressing or ignoring the demands of the social movement. In particular, we focus on how the network of social connections between the groups in society affects protest participation and the government's actions.

The issues we investigate here are especially relevant to contemporary politics as a myriad of mass protests occur worldwide. Even though these social movements happened in countries as diverse as Iceland, Egypt and the USA, they all have in common a large use of mobile phones and social media for communication among its members. Consequently, these movements also share organizational characteristics such as horizontality and heterogeneity of demands, because the cost of organizing a collective action with the new available communication technology was drastically reduced and the dependence on traditional groups of interest, like labor unions and political parties, diminished. Castells (2013)

In order to explore this questions, we study a sequential game played between a government facing electoral incentives and citizens organized in decentralized groups defined by a common preference for the public policy, as in traditional models of probabilistic voting Lindbeck e Weibull (1987), Persson e Tabellini (2000). The citizens can influence public policy by protesting in two periods and voting, and the government can affect the second period protest by repressing the movement with violence or by conceding to some organized groups.

The mechanism of protest formation studied in this paper draws on insights from Granovetter (1978) and Passarelli e Tabellini (2013), who formalize and extend the former's ideas. As the probability of being pivotal is vanishingly small in large societies, individuals recognize that their presence at a protest is irrelevant to the success or failure of the movement. Hence, people *do not* protest for instrumental reasons such as the expected change of public policy towards their bliss point. Instead, individuals feel aggrieved when they judge the government's policies do not attend their group specific demand or when they are violently repressed. The protest participation is then based on the psychological reward of joining other people in public displays of frustration.

Also alike Passarelli e Tabellini (2013), we suppose that agents' utility is increasing in the number of protesters from their own group, but we extend their framework to allow for both positive and negative externality of participation *across* groups. This simple extension yields a much richer setting, in which it is possible to study issues of how group identity and leadership affect protest formation. We explore such questions building on the model of heterogeneous peer effects by Ballester et al. (2006), Ballester et al. (2010), using graph theory insights to understand the protest participation in equilibrium.

The government, after observing the first period of protest, chooses between conceding, repressing or ignoring the protesters. Even though our model can be directly applied to dictatorships, we focus on a democratic setting, in which agents can punish government actions by voting on the opposition. Hence, the incumbent must trade-off the political cost of repression with the political cost of protests, as both affect his popularity among voters. Concessions are also politically costly because the incumbent can alternatively use his budget for campaign expenditure. The costs are weighted against the benefits of repressing or conceding to a subset of the organized groups, as both actions may reduce the number of participants on future protests.

We first analyze the model in a completely general network of influence, in which we allow for asymmetrical and both positive and negative participation externalities across groups. In this setting, we restrict the government's decision of concession by allowing it to make concession to at most one group. Our results extend to more general setting and permit the direct application of the results from Ballester et al. (2006) to identify who is key group on the protest's network, i.e., the group whose removal from the network maximally reduces aggregate protest participation in the next period.

With this result in hands, we prove that there is a threshold such that the government will find it optimal to repress the protests if and only if the key group's centrality in the network of influence is below that level. When the key group is very central to the protest organization, the government will find optimal to concede to them instead of repressing the movement, since its withdrawal from the network will significantly diminish aggregate participation. In the context in which groups only influence the others *positively*, the interpretation of this result is that more horizontal protests, without strong leaderships, are more likely to be repressed by the government.

As a direct application of this general result, we study the effects of the recent communication revolution on the pattern of protest participation and the trade-off between concession and repression. We show that as the introduction of internet and mobile phones reduces the cost of communication for less organized groups, it wakens the movement's dependence on more organized groups. In other words, the centrality of the organized groups is reduced when communication gets cheaper, which makes repression more likely. On the other hand, improvements on the communication technology also reduces the incumbent's probability of reelection, making the effect of communication technology in protesters' utility ambiguous.

Next, we explore the idea that the government can use radical groups to weaken social movements. First, we show that when the incumbent is lucky enough so there is a radical unpopular group participating in the protests massively, the government will eventually ignore the protests. Also, as the level of unpopularity of this radical group increases, so does the incumbent's chance of reelection. However, even if the presence of a radical group is harmful to the social movement, it is not always the case that there is one participating. Hence, we explore three different ways in which the government may incite the participation of radical groups.

First, we show that if there is a violence-prone radical group in society, i.e. a group that is unpopular and participate more heavily when they are repressed, the government may have an additional incentive to repress the protests. Second, we show that when there are two radical groups from the opposite sides of society's ideology spectrum, the government may encourage the participation of one radical group by conceding to the other. The absence of the latter group on future protests will stimulate the participation of the remaining radical group, which may reduce the aggregate participation of the moderate groups in society. Finally, we extend the model to explore a situation in which the government can invest in propaganda against a particular group, changing the perception of the remaining groups about it.

This paper is related to several literatures. First, in the protest literature, the papers that more closely relates to ours is Passarelli e Tabellini (2013), whose debts and differences were already discussed, and the seminal contribution of Suzanne Lohmann (1993), Lohmann (1994), Lohmann (1994), which characterizes protests as a signalling phenomenon. Agents choose to protest to inform the government of their preferences, in a process that partially aggregate the information on the economy about an unknown state of the world. Her model, however, cannot explain the obvious fact that, many times, governments do try to hinder protests in any way possible. In her framework, protests are harmless to the government, which is benevolent. It only affects the government choices because the number of people protesting is a sufficient statistic about the true state of the world.

Another difference from our modelling is the reason why individuals protest. She presupposes that the agents' protest decision is based on a calculation of the expected value of *their* participation on the policy outcome, while we assume that individuals protest because they are aggrieved and derive psychological rents from joining public displays of discontent against the government Passarelli e Tabellini (2013). The grievance theory is traditional in the political science and social psychology literatures, with initial contributions dating back to Gurr (1970) and Berkowitz (1972).

This approach is very useful because it allows us to explain why people do not simply free ride on others' protest participation, as predicted in the classic contribution of Olson (1965). There is abundant evidence, both theoretical and empirical, that collective action cannot be sustained only through pivotal reasoning Myerson (2000), Feddersen (2004). Moreover, there is growing lab evidence that individuals do have a taste for punishing social norm violations, in our case unfair policies, even when incurring in non-trivial private costs to do so (Fehr and Gatcher, 2000).

Second, by allowing groups' participation in protests to generate both positive and negative externalities in each other, we contribute to the identity economics literature Akerlof e Kranton (2000). Considerations about identity are also traditional in the social psychology and sociology literatures, and we formalize and expand the idea that intense group identity facilitates groups in overcoming the collective action problems Van Stekelenburg e Klandermans (2013).

Our model also relates to the vast literature that studies how disenfranchised citizens may interact with non-democratic governments Acemoglu e Robinson (2005), Bueno de Mesquita et al. (2003), Haggard e Kaufman (2012), McAdam et al. (2001). In a way, understanding the costs of protests to an authoritarian government is more straightforward: the dictatorship fears to be overthrown by an organized social movement and may choose to give concessions or repress it to avoid that risk. On a democratic setting, on the other hand, both the incentives of protesters and the effects protests have on the incumbent party are less clear. This paper contributes to this discussion by modelling how electoral concerns gives incentives to governments do deal with protests.

The rest of the paper is organized as follow: in section 2 we introduce the model and discuss the main hypothesis underpinning it. Section 3 presents results concerning the identity of the key group of the network and characterizes the government's trade-off. We also explore two applications of the model concerning the role of leadership. Section 4 applies and extends the general model to show how governments can use unpopular groups to weaken the protests. Section 5 concludes.

2
The Model

There is a set $\mathbf{N} = \{1, 2, ..., N\}$ of groups in the society, each consisting of a measure one continuum of agents. The groups are defined by a common preference of their agents for a public good that interests only the groups' members, which is henceforth denominated *a concession*. Each individual of each group has an idiosyncratic aggrievement level toward the government that depends on whether the group receives or not a concession or was repressed. The agents can influence political outcomes in two manners: they can choose to participate in two rounds of protests and they can cast a vote to their preferred candidate.

There are two political agents, the incumbent and a single opponent on elections. The incumbent bears the political cost of protests and chooses, between the two rounds of protests, whether to make concessions to any subset $S \in 2^{\mathbf{N}}$ of the groups or to repress the protests in $t = 1$. We suppose that, in the absence of repression, any group receiving a concession has zero protest participation in the second period. After a possible second round of protests, elections are held between the incumbent and a single opponent. On that stage the incumbent's chance of reelection is affected by the choices of concession and repression, the strength of the protests, and agents' aggrievement level.

The next three subsections explain the details of each period of the game, that has the following timing:

1) Each citizen of each group in $\mathbf{N}$ decides whether to protest.
2) The incumbent decides to which subset of groups $\mathbf{S} \in 2^{\mathbf{N}}$ he will give a concession and whether he represses the protest in 1 or not.
3) Each agent from the groups that did not receive a concession, $J \in \mathbf{N} \setminus \mathbf{S}$, decides whether or not to protest.
4) Election between the incumbent and a single opponent is held.

2.1
Protests

The mechanism of protest formation in our model is based on the modelling of Passarelli e Tabellini (2013), which formalize and extend insights by Granovetter (1978). As the probability of being pivotal is vanishingly small in large societies, individuals recognize that their presence on the protest is irrelevant to the success or failure of the movement. Hence, agents do not base their decision of protesting in being pivotal to the policy changes that can be accomplished through the social movement. Instead, individuals feel aggrieved when they judge the government's policies are below the level they feel entitled to. Then, if an individual is aggrieved and participates in a protest, he receives a psychological reward for joining other people in a public display of the frustration caused by the policy Passarelli e Tabellini (2013). This formulation has the advantage of overcoming the free-rider problem, which is the main issue for collective action to succeed (Olson, 1965).

Decision of Protesting

Each agent of group J make the decision of protesting based on their aggrievement level, the cost of participation and the expected participation of other individuals. Agent's i aggrievement is given by

$$a^{iJ} = \delta^J r + \gamma^J (1 - c^J)(1 - r) + \epsilon^{iJ}. \tag{2-1}$$

The variables r and c^J are the endogenous binary variables that represent, respectively, if there was repression ($r = 1$) and if group J received a concession ($c^J = 1$).[1] The parameters δ^J and γ^J measure how group's J aggrievement level is affected by repression and the absence of concession. Note that we assume concessions reduce aggrievement only when there was no repression. ϵ^{iJ} is the idiosyncratic part of aggrievement and is uniformly distributed on the interval $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$. We assume that $\delta^J > 0$, which means individuals feel more aggrieved - and are therefore more willing to protest - when they are repressed.

On the other hand, we assume that the cost of participation increases when the government represses the protest. Formally, we assume that participation cost is given by

$$\mu^J = \mu_0 + r v^J \tag{2-2}$$

where $\mu_0 > 0$ is the initial cost of opportunity shared by all groups and $v^J > 0$ measures how costly violence is for individuals of group J.[2]

Finally, we assume that individuals care about who and how many people they are protesting with. Individuals are influenced by participants of their and of other groups. This influence needs not be symmetrical or positive and

[1] The variable r has no superscript because we assume the incumbent can either repress all or none of the groups.

[2] There are many different ways for justifying why repression increases the participation cost. For instance, repression may signalize that the government is violent and will repress future protests again Glaeser e Sunstein (2015). Also, repression on the non-democratic politics literature has widely been assumed to disband the protests Acemoglu e Robinson (2005). Our approach is a generalization as the same result could occur provided that v^J is large enough for every group.

it is represented by a weighted network $\mathbf{G}$. The adjacency matrix of this network is the NxN matrix G such that $G_{ik} \in [-\kappa, \kappa]$ and $G_{ii} > 0$, with $\kappa > 0$. Therefore, protest participation presents intra-group complementarity and either complementarity or substitutability across groups.

Let $\mathbf{x} \equiv (x_1, ..., x_N)$ be the vector of protest participation. The agent's utility in protesting is given by

$$u^{iJ}(\mathbf{x}, a^{iJ}; G) = a^{iJ} - \mu^J + \sum_K G_{JK} x_K \tag{2-3}$$

The entry JK of matrix G capture the participation externality group K individuals generate on the payoff of participation from individuals of group J.

We derive now the vector of equilibrium participation. To do so, we first impose conditions such that the solution is unique and interior, i.e. each groups has a positive mass of agents participating and a positive mass of agents *not* participating.

Lemma 1 *Let* $\theta^K = \mathbb{E}[a^{iK}|r, c]$. *Also, define* $\bar{\mu} = \max_J\{\mu + v^J\}$, $\bar{\theta} = \max_J(\theta^J) = \max_J\{\delta^J + \gamma^J\}$, *and* $\min_J(\theta^J) = \min_J\{\gamma^J\}$. *G is a N**x**N matrix such that* $G_{ik} \in [-\kappa, \kappa]$ *and* $G_{ii} > 0$.
(a) *The equilibrium participation vector is interior if*
(i) $\underline{\theta} + \frac{1}{2\phi} > \bar{\mu} + \kappa(N-1)$, *and*
(ii) $\bar{\theta} - \frac{1}{2\phi} < \mu_0 - \kappa N$.
(b) *The equilibrium participation vector is unique if* $\phi\kappa < \frac{1}{N}$, *which is implied by* ***(a)***.[3]

Let $\mathbf{e_j}$ be the j'th canonical vector. The marginal agent of group J is then $\tilde{a}^J = \mu^J - \mathbf{e_j}' G\mathbf{x}$. Thus, for each J, $x_J = Pr(a^{iJ} \geq \tilde{a}^J)$. Since aggrievement is uniformly distributed around θ^J and the solution is interior, this is equivalent to

$$x^J = \frac{1}{2} + \phi\left(\theta^J - \mu^J + \mathbf{e_j}' G\mathbf{x}\right)$$

The above expression can be rewritten in matrix form as $\mathbf{x} = \phi G\mathbf{x} + \boldsymbol{\alpha}$, where $\boldsymbol{\alpha} \equiv (\alpha_1, ..., \alpha_N)$ and $\alpha_J = \frac{1}{2} + \phi(\theta^J - \mu^J)$. Thus, the vector of equilibrium participation, conditional on network G, is given by

$$\mathbf{x}(G) = (I - \phi G)^{-1}\boldsymbol{\alpha} \tag{2-4}$$

where I is the identity matrix. This result holds if and only if the equilibrium participation is interior and the matrix $(I - \phi G)$ is invertible, which is

[3] It is worthwhile noting that these conditions are not necessary, and the interiority one is easily relaxed by adding more structure to G.

guaranteed by lemma (1). Note that $(I-\phi G)^{-1} = \sum_{k=0}^{\infty} \phi^k G^k$, which explains why we must have $\phi\kappa < \frac{1}{N}$: when $\phi\kappa$ is too high the complementarities or the diffusion on the network are too strong and the power series diverges.

As first noted in Ballester et al. (2006) in a linear-quadratic model of social interaction, the vector of equilibrium participation in (2-4) coincide with the Bonacich network centrality measure Bonacich (1987), a n-dimensional vector capturing the centrality of each node on a given network.

Definição 2.1 *Let $M(G,\phi) = (I-\phi G)^{-1} = \sum_{k=0}^{\infty} \phi^k G^k$. The Bonacich centrality of the network G with discount ϕ weighted by α is the vector*

$$\boldsymbol{b}_{\boldsymbol{\alpha}}(G,\phi) = M(G,\phi)\cdot\boldsymbol{\alpha} \tag{2-5}$$

The original definition of Bonacich centrality concerned unweighted undirected networks. When the vector $\boldsymbol{\alpha} = \mathbf{1}$, this measure counts all the walks starting from a given node, with walks of length k weighted by ϕ^k. In the context of directed weighted networks, instead of measuring the number of walks, the Bonacich centrality captures the sum of influence a given node *receives* from the rest of the network directly and indirectly. The matrix $\phi^k G^k$, for instance, measures the influence the nodes exert in each other indirectly through the influence they exert in a sequence of other $k-1$ nodes. Finally, a vector $\boldsymbol{\alpha} \neq \alpha\mathbf{1}$ must be taken into account when some groups are more active than others because of characteristics like lower opportunity cost.

We provide some comparative statics results regarding the aggregate participation when the network is of complementarities only. The comparative statics are more complicated when there are also negative participation externalities, and we explore this issue on the next sections.

Proposição 2.2 *Suppose that G is a matrix of complementarities. Then, the following results hold:*

(a) Aggregate protest participation increases with ϕ, γ^J, δ^J for any J and any entry of G.

(b) Aggregate protest participation decreases with μ_0 and v^J for any J.

(c) If the network G is strongly connected, then not only aggregate protest participation increases (decreases), but the participation of each group increases (decreases).[4]

Because an increase in γ^J, δ^J raise group's J average aggrievement and a decrease in μ_0, v^J lower their opportunity cost of protesting, the participation

[4]Strongly connected means that you can reach every nodes from each node in the network. As the network may by directed, a path is constituted by non-zero elements of the matrix G.

of this group will increase. Since the network is of complementarities only, the larger presence of group J members encourages further participation of the groups connected to J, explaining the effect of these variables in aggregate participation.

When we increase an entry of the network G, it means that a link between two groups becomes stronger (or we create a new link on that network). As in the previous paragraph, with a network of complementarities, this helps disseminate the influence of one group on the others, increasing aggregate participation. The interpretation of the comparative statics of ϕ is similar but more interesting. This parameter measures the density of the aggrievement's distribution in each group. When ϕ is large, it means that the individuals from each group are quite homogeneous and a marginal increase in participation from members of any other groups implicate a more intense response in participation from the connected groups. Thus, the parameter ϕ has an interpretation of intensity of diffusion of influence on the network G, which implies that an increase in its magnitude makes the diffusion of positive participation externality more intense to nodes farthest away from each other.

Finally, item (c) simply states that if every node of the network can be reached from each other node in the network, the effects are not only aggregated, but are also valid for each group. This happens because strong connectedness guarantees that the influence of an increase in participation from one group will eventually reach all the other groups in the network.

2.2 Political Competition

There is a continuous public good Q which benefits all the agents in the economy. We assume that each agent from a given group J has homogeneous preference for Q, given by the strictly concave utility function $W^J(Q)$. We assume the politicians can commit to their platforms on the general interest good Q, but they cannot commit to group concessions for the next period. This assumption lies on the fact that there are a large number of small interest groups, each with a small probability of overcoming the collective action problem and participate on protests after the elections.

There are two politicians, the incumbent (I) and the opponent (O). They derive utility from an ego rent of being in office, denoted by R. If $Pr(P)$ is the probability of the politician P win the election, his utility is given by $V^P = Pr(P)R$. The probability of winning the election is endogenous, depending on three factors: the platforms announced by the candidates, the heterogeneous aggrievement towards the government and the popularity of the candidates.

We build on the models of lobbying by Grossman e Helpman (2002) and Persson e Tabellini (2000) assuming that the popularity of the candidate is given by a stochastic term, ξ, determined just *after* the platforms' announcement, whose mean can be affected by the environment and the politicians' actions. We normalize the opponent's popularity to zero and assume the incumbent's to be

$$\xi = \tilde{\xi} - f(r, \mathbf{S}, \mathbf{x}^1, \mathbf{x}^2) + e \tag{2-6}$$

where $\tilde{\xi}$ is a random variable distributed in accordance to a strictly increasing and symmetric[5] CDF H, representing the stochastic term of popularity. The function f maps how protests in both periods, as given by the vector of participation $\mathbf{x}^1$ and $\mathbf{x}^2$, and repression (r) affects the popularity of the incumbent, given his choice of concessions $\mathbf{S} \in 2^{\mathbf{N}}$. Finally, e is a measure of expenditure in campaign advertisement the incumbent can use to increase his popularity.

In traditional models of lobbying, donations for campaign advertisement are used by special-interest groups to influence politicians. Here, it sheds light on a possible channel through which the cost of the concessions, given by the function $C(.)$, affects the incumbent's utility. We assume that the expenditure on advertisement by the incumbent is limited to

$$e \leq \bar{y} - C(\mathbf{S}) \tag{2-7}$$

where $\bar{y}$ is the (exogenous) budget of the government and $C(0) = 0$, $C' > 0$. Hence, expenditure in group-specific public goods curtails the incumbent's capacity of making campaign advertisement to enhance his popularity.[6]

We assume that agent i from group J cast his vote to the incumbent if and only if

$$W^J(Q^I) - W^J(Q^O) + \xi - a^{iJ} \geq 0$$

where a^{iJ} is again the individual's i aggrievement, which accounts whether or not the group received a concession or if it was repressed, as in equation (2-1).

We provide the following lemma that asserts the convergence of the two

[5]This assumption is dispensable and we use it just for algebraic convenience.

[6]Alternatively, we could make small changes in the problem's formulation to contemplate the possibility that politicians are not only office-seekers but also rent-seekers, with the amount $\bar{y} - C(\boldsymbol{q})$ being the rent they extract from office. We stick to the former interpretation as the latter is more difficult to reconcile with the utility function we proposed for the politicians, since on the second period in office they do not take into account the utility from rent-seeking, only the ego rent R.

politicians' platforms[7]

Lemma 2 *Both candidates announce the same platform $Q^{I*} = Q^{O*}$ and the incumbent's probability of reelection is given by*

$$Pr(I) = H\left(-r\frac{1}{N}\sum_{J}\delta^J + \bar{y} - f(r, \mathbf{S}, \mathbf{x}^1, \mathbf{x}^2) - C(\mathbf{S})\right) \tag{2-8}$$

where $H(.)$ is the CDF of $\tilde{\xi}$.

The term $\frac{1}{N}\sum_J \delta^J$ stands for the average increase in aggrievement across groups caused by repression, and it represents an additional cost of repression not captured by the political cost function f. For the sake of tractability, the political cost function is assumed to be linear:

$$f(r, \mathbf{S}, r, \mathbf{x}^2) = \begin{cases} \frac{\zeta}{N}\sum_{J\notin\mathbf{S}} x_J^2(G_{-S}, r) & \text{if } r = 0 \\ \frac{\zeta}{N}\sum_{J\in\mathbf{N}} x_J^2(G, r) & \text{if } r = 1 \end{cases} \tag{2-9}$$

where $x_J^2(G)$ is the equilibrium participation of group J in round 2 of protest given the social network G, repression choice r and, again, $\mathbf{S}$ is the subset of groups the government chose to concede to. Finally, ζ is a positive parameter capturing the marginal costs of protests.

Function (2-9) asserts that the government suffers a political cost proportional to the mass of protesters on the second period. This value varies when there is either repression or some level of concession. Note that adding a negative impact of first period protest participation is irrelevant to the government's choice since it is a sunk cost, and that the government will never give concessions and repress because we assumed that groups repressed ignore their concessions.[8]

The hypothesis that the incumbent's probability of reelection decreases with the size of the protest can be rationalized in different ways. For instance, protests reduce the welfare of the society as it can cause damages to the city structure and disturb citizens' work and leisure activities. Then, the incumbent's popularity could be affected is he is even partially blamed by this loss of welfare. Also, political beliefs from the society as a whole may shift through social interaction with passionate protesters. Finally, it is possible that the number of protesters must be big enough so they get media coverage, making the issues raised by the social movement salient to the public opinion.[9]

[7]We add the part γ^J of the average aggrievement, which represents the aggrievement caused by lack of concessions, to the cost function $C(\mathbf{S})$ with no loss of generality.

[8]The results are robust to adding an additional cost of repression proportional to the mass of protesters on $t = 1$.

[9]Madestam et al. (2013) shows that the size of protests affect negatively the probability

The incumbent maximizes the probability of reelection, given by equation (2-8). Since H is strictly increasing, this is equivalent to minimizing the loss function[10]

$$L(r, \mathbf{S}) = r \sum_J \delta^J + n \cdot f(r, \mathbf{S}, \mathbf{x}^2) + C(\mathbf{S}) \quad (2\text{-}10)$$

Because the game is solvable by backward induction, we henceforth ignore this stage of the game (which is the last one) and assume that the government, when facing protests in period 1, chooses between concession and repression by minimizing the loss function (2-10).

2.3 Government Action

As we have discussed, the government end goal is to reduce protest participation on the second round of protests and it is capable of doing so by conceding to any subset of groups or by repressing all or none of the groups. The incapacity of discriminate repression among groups conveys the fact that, in the context of mass protests, individuals are often mingled with people from different groups and there is no obvious signal from which group they belong to.[11]. Thus, giving the loss function (2-10), the government will choose to repress the protests if and only if

$$\sum_J \delta^J + \zeta \sum_{J \in \mathbf{N}} x_J^2(G, r = 1) < \min_{\mathbf{S} \in 2^{\mathbf{N}}} \left\{ \zeta \sum_{J \notin \mathbf{S}} x_J^2(G_{-S}, r = 0) + C(\mathbf{S}) \right\} \quad (2\text{-}11)$$

Condition (2-11) makes it clear that the end goal of the government is to reduce protest participation on the second period. There are two ways it can do so: first, it can make costly concessions that remove the conceded groups from the network. This may reduce aggregate participation both directly and indirectly, because groups generate participation externalities in each other. Second, the government can repress the protest, which is also costly, as captured by the increase in average aggrievement, $\sum_J \delta^J$. This option may reduce aggregate participation if it significantly increases the cost of protesting.

Conditional on not repressing, the problem of the government can be solved in two stages. Fix a number of groups to receive concession, k, and find

of reelection of the politicians or parties targeted by the protesters with evidence from the Tea Part Movement, in the US.

[10]Since C is an arbitrary function, we can multiply the loss function by the number of groups N and incorporate this term in C with no loss of generality.

[11]A more realistic and general assumption would be that the government can observe the group each agent belong to with some noise. Then, the government could choose to repress an individual group but it wouldn't be able to repress every member of this group. Besides, there would be individuals from other groups being repressed when their signals were misinterpreted. Our assumption is a particular case of this framework, when the individuals' signals are infinitely imprecise.

the subset of player $\mathbf{S}$ of size $|\mathbf{S}| = k$ that minimizes the protest participation on the subsequent period. Repeat this process for each possible group size to encounter the optimal subset of players to concede to. The latter step is trivial while the first one has been analyzed in Ballester et al (2010).[12] This problem is analytically intractable in all of its generality, so later sections are dedicated to the development of results while imposing further structure to the model. We nevertheless establish a basic result on the following proposition.

Proposição 2.3 *If G is a network of complementarities and $\delta^J > v^J$ for every group J, then the government will never repress the protests .*

The condition $\delta^J > v^J$ on the proposition above simply says that the increase in aggrievement caused by government's repression is higher than the increase in the cost of protesting for every group. Therefore, disregarding participation externalities, repression increases the participation of every group. The fact that all the participation externalities are non-negative guarantees that the increase in participation of any group do not deter the participation of member from the other groups.

Now, if there were some groups generating negative participation externality, proposition (2.3) might not be valid anymore, because the increase in participation of one of these groups may generate such a negative effect to the other groups' members that their participation is actually reduced after repression. We explore this situation on section 4. Another feature of this result is that it holds for any cost function C. On section 3 we add assumptions about this function, allowing us to study the relation between concessions and repression.

[12]They prove this problem is NP-hard, meaning that solving it computationally is at least as hard as the hardest NP problem.

3
The Key Group and Leadership

In this section we analyze the role of leadership in protests and how it affects the trade-off between concession and repression faced by the government. We make no restriction to the externalities groups can generate o each other and define the cost of concession function

$$C(S) = \begin{cases} 0 & \text{if } |S| \leq 1 \\ \infty & \text{if } |S| \geq 2 \end{cases} \tag{3-1}$$

where $|S|$ is the number of concessions chosen. Hence, whenever there is a group whose withdrawal from the network diminishes aggregate participation, the government chooses between repression and concession to one group only. The group the government finds it optimal to give a concession to will be called *the key group*, a terminology employed in Ballester et al. (2006), Ballester et al. (2010). If we suppose that each group generates only non-negative influence in each other, we call the key group the *leader* of the protest.

In the first subsection we identify, on a general network setting, what is the relevant measure to identify the key group, a result that simplifies the analysis without much loss of generality from cost functions different than $(3-1)$.[1] We also explore the effect of leadership on the likelihood of repression. Next, we give two examples of situations in which our model may be applied.

3.1
Identity of the Key Group and the Role of Leadership

Let G be a network such that $G_{ii} > 0$ and $G_{ij} \in [-\kappa, \kappa]$ for $\kappa > 0$. With the cost function given by (3-1), we have to develop a way to define which is the key group in G. Since the government may choose at most one group to concede to, substituting equation (3-1) on (2-11) implies that the government represses iff

$$\Delta + \zeta \sum_{J \in \mathbf{N}} x_J(G, r=1) + \leq \zeta \cdot \min_{K \in \mathbf{N}} \left\{ \sum_{J \neq K} x_J(G_{-K}, r=0) \right\} \tag{3-2}$$

where $\Delta \equiv \sum_J \delta^J$.

[1]Remember that the government's program can be solved in two steps, we are just assuming that the best cost benefit to the government is with one group, but the same analysis could be done to any arbitrary number of groups.

Therefore, conditional on not repressing, the government will give a concession to the group that mostly reduces aggregate participation on protests, the key group. Clearly, this could be done by simply removing each group and calculating the aggregate participation on each sub-graph G_{-J}, choosing the lowest one. This approach has two drawbacks: first, it can be computationally intense for large complex networks. Relatedly, it is more practical to have a statistic to determine the key group depending exclusively on the original network. Second, this approach is silent on what makes the removed group more important than the others.

Alternatively, we use a measure of network centrality proposed by Ballester et al (2006, 2010) to define who is the key group. The model in this section is analogous to the one developed there to pursue the optimal crime deterrence policy when criminal activity is complementary on a graphical game. There, a planner needed to withdraw the most influential criminal from the network in order to minimize aggregated crime. They show that in spite of the most central player in the Bonacich measure being also the one with the highest criminal activity, it may be suboptimal to withdraw him from the network.

The reason why the Bonacich centrality is the relevant measure of participation and fails to determine the leader is that it takes into account only the player benefit in participating. Ballester et al (2006, 2010) then presented the *intercentrality measure*, which captures the externality generated by the group's participation in the protest.

Definição 3.1 *Let* $\boldsymbol{\alpha}$ *be the vector of characteristics of each group and* M *the matrix defined in (2-5). The intercentrality measure of group* i *is given by*

$$d_{\boldsymbol{\alpha}}(G,\phi)_J = X(G, r=0) - X(G_{-J}, r=0) = \frac{b_{\boldsymbol{\alpha}}(G,\phi)_J \cdot \sum_{K=1}^{N} M_{KJ}(G,\phi)}{M_{JJ}(G,\phi)} \tag{3-3}$$

where $X(G, r=0) \equiv \sum_J x^J(G, r=0)$ *is the aggregate participation without repression.*

While the second term on (3-3) shows that the intercentrality measure is exactly the difference between aggregate participation with and without the group J, the third term shows that the intercentrality measure counts all the paths of influence of the network that pass through J. The term $b_{\boldsymbol{\alpha}}(G,\phi)_J$ is the influence that arrive on J directly and indirectly, as already discussed, and the term $\sum_{K=1}^{N} M_{KJ}(G,\phi)$ captures the impact of J on the other groups. Finally, this two terms double count the self-loops, given by $M_{JJ}(G,\phi)$, explaining why we must divide for it. Besides having a clear interpretation, the third term is instrumentally useful, as it allows us to focus on a measure that depends exclusively on the original network.[2]

Since the group with the highest intercentrality measure is, by definition, the one with the largest number of paths of influence passing through him, it is fairly intuitive that this group should be the networks' leader. As a matter of fact, it is straightforward to see that

$$\max_i \{d_{\boldsymbol{\alpha}}(G,\phi)_i\} = \max_i \{X(G,0) - X(G_{-J},0)\} = \min_i \{X(G_{-J},0)\}$$

since $X(G,0)$ is a constant from the point of view of the optimizer. Thus, because $X(G_{-J},0) \equiv \sum_{K \neq J} x_K(G_{-J})$ we have the following proposition.

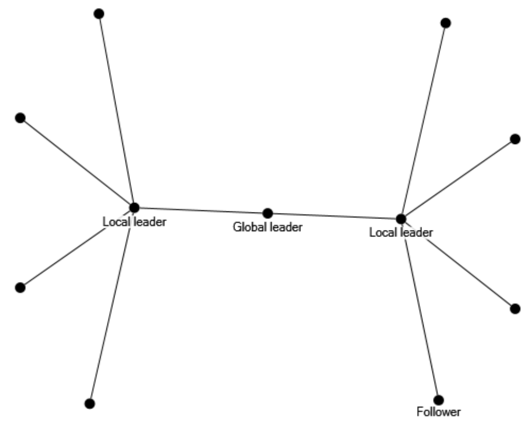
Proposição 3.2 *Conditional on not repressing, the government makes a concession to group J if and only if $J \in arg \max_{i \in \mathbf{N}} \{d_{\boldsymbol{\alpha}}(G,\phi)_i\}$ and $d_{\boldsymbol{\alpha}}(G,\phi)_J > 0$.*

The condition that $d_{\boldsymbol{\alpha}}(G,\phi)_J > 0$ only states that there is a player whose withdrawal from the network reduces aggregate participation.

It is fair to ask whether the question of who is the key group is an obvious one. To show that this is not the case we explore the following example, loosely based on an application from Ballester et al. (2010).

Local vs Global Leader

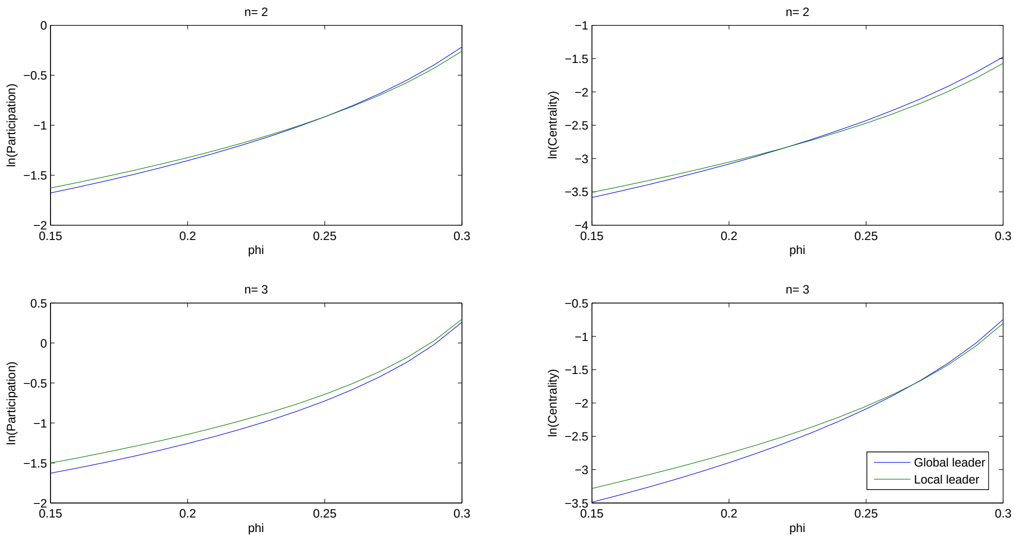
Figure 3.1: Local vs Global Leader - Network



We study the network on Figure 1, consisting of three different types of groups. There are two kinds of leaders, the global leader, which connects the two big components of the network, and two groups we call the *local leader*. The latter have that name because they are the sole connection to the participation

[2]For a proof of the second equality see the demonstration of Theorem 3 in Ballester et al (2006).

Figure 3.2: Local vs Global Leader - Participation and Intercentrality



of $2n$ other groups, which we name the *followers*. We also assume that each pair of connected groups generate a positive externality of participation with weight 1, and each pair of disconnected groups generate no *direct* externality in each other. Figure 1 shows this setting for $n = 3$.

The network in figure 1 has two interesting features. First, as we show below, even restricting the analysis to a symmetrical setting in which the only heterogeneity among groups is their location on the network, the question of who is the key group is not obvious. On the one hand, the local leader is responsible for the connection of a lot of groups that without him would not exert any influence in each other. On the other hand, the bridge can transmit the complementarity through all the network, generating positive externality between the two components of the network. Second, it illustrates the importance of the parameter ϕ, which measures the capacity of diffusion in the network.

We answer the question of who is the key group with a numerical example of the model, calculating the participation and the intercentrality of both the bridge and the local leader group, setting $\alpha^J = 0.1$ for every group and varying ϕ and the number of followers, n. The results are displayed on figure 3, with the plots on the left hand side of the image showing the log participation of the global leader and the local leader, respectively. The graphs on the right hand side of the image show the log of the centrality of each type of player. The graphs on the top display these values for two followers while the figure on the bottom show it for three.

When n=2 the participation of the global leader eventually becomes bigger than the participation of the local leader as ϕ increases. The same happens with the intercentrality of the global leader, but, interestingly, the global leader becomes more central *before* it starts to participate more. This pattern is even stronger for $n = 3$. In this case, the local leader protests more for every level of ϕ that generates an interior equilibrium solution, but the global leader eventually becomes more central than the local leaders.

As in Ballester et al. (2010), higher levels of ϕ gives more centrality to the global leader because as ϕ grows, the impact of indirect externalities - those between unconnected groups - increases. We have a further interpretation of the result as, in our model, ϕ stands for the dispersion of the distribution of aggrievement in each group. As the density of the distribution of aggrievement becomes bigger, each additional protester from other groups swings a larger mass of agents into protest participation. This is the same intuition of probabilistic voting models as Lindbeck and Weibull (1986).

***

Now that we have shown which group receives a concession, we determine under which conditions the government will choose whether or not to repress the protests. Let J^* be the key group of the network G. We define the aggregate number of protesters with network G and repression decision r by $X(G, r)$. The following proposition describes the solution to the government's program.

Proposição 3.3 *The government will choose to repress the protests if and only if*

$$d_{J^*} < X(G, 0) - X(G, 1) - \frac{\Delta}{\zeta} \tag{3-4}$$

The inequality (3-4) is very helpful in understanding the costs and benefits of concession and repression. If the protest owes most of its participation to a single group, the key group, concession becomes more likely than repression. This is captured by the intercentrality measure d_{J^*} on the right-hand side of the expression above. Then, a social movement with a clear leadership is less likely to be repressed than a more horizontal movement as the benefit of concession, measure by its effectiveness in reducing aggregate participation, is larger.

The firs term on the left-hand size, $X(G, 0) - X(G, 1)$, measures the effectiveness of repression as a mean to reduce aggregate participation. The larger this difference, which measures the deterrence in participation caused by repression, the more likely repression it is to occur. As opposed to concession, which we assumed to be costless for one group, repression is costly, as captured by $\frac{\Delta}{\zeta}$. Δ is the sum of increase in aggrievement caused by repression, capturing the negative effect of the incumbent's choice on his chance of reelection. Thus, it is possible that even if the increase in the cost of protesting is much higher than the increase in aggrievement, resulting in a significant deterrence effect, repression will not worthwhile when individuals feel aggrieved by repression. Finally, the term is divided by ζ, which indicates that as the political cost of protests increases, the likelihood of repression increases.

Besides the comparative statics on ζ, discussed above, we can also add further comparative statics results on the likelihood of repression. In the proposition above, we take an increase in the likelihood of repression to be increases on the function $X(G,0) - X(G,1) - \frac{\Delta}{\zeta} - d_{J^*}$, given by inequality (??) above, caused by increases on the parameters of the model.

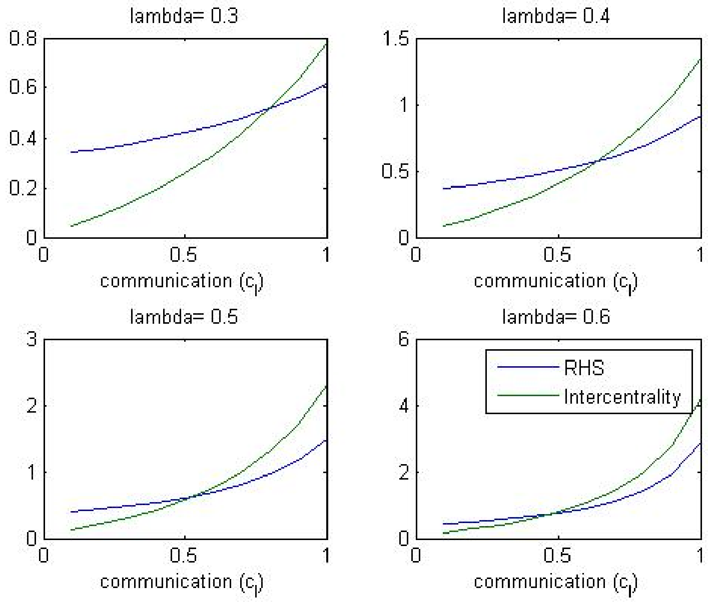
Proposição 3.4 *Suppose G is a network of complementarities only. Then,*

(i) If $\mathbf{v}$ *decreases or* $\boldsymbol{\delta}$ *increases, the likelihood of repression decreases.*
(ii) If J^* *is the key group of a non-negative network G, an increase on* G_{KJ^*} *and* G_{J^*K} *for any* $K \neq J^*$ *reduces the likelihood of repression.*

The result of item (i) is intuitive. A decrease in v^J for one group means that repression partially loses its deterrence effect on this particular group. The fact that we have a network of complementarities implies that the increase in participation of group J does not discourage other groups members' participation, making repression a less effective measure of reducing protests. An increase in δ^J also reduces the benefits of repression, since individuals of that group become more aggrieved and willing to protest after repression, but also affects the cost of protests, since more aggrieved voters will have a higher probability of ousting the incumbent of office. Then, if we increase $\mathbf{v}$ and $\boldsymbol{\delta}$ for the same amount on the same direction, the benefit of repression remains constant, but the cost of repression increases, reducing the likelihood of repression.

Item (ii) states that when the leader becomes more important by increasing its influence on some other groups, the likelihood of repression is reduced. This goes in the direction, as already commented, of the general result that protests with strong leaderships are less likely to be repressed than more horizontal movements. Interestingly, when the network G is not positive, meaning that some groups exert negative participation externality on others, making a tie between a leader and other group "more positive" does not imply that the leader's intercentrality will increase. Therefore, there is no guarantee that such a comparative statics is monotone, and as a matter of fact we explore such an example on the next section.

Figure 3.3: Communication Revolution and Repression



3.2
Example: Communication Revolution and Mass Protests

In this subsection we use our model to investigate how the recent communication revolution affected the organizational structure of protests and how it affects the government decision. On the past few years, a myriad of mass protests occurred worldwide in countries as diverse as Brazil, Iceland, Egypt, and the USA. Even though the motivation of the protests and the socio-economic background of these countries are completely different, they all have in common a large use of mobile phones and social media for coordination.

This feature crucially affects the groups of interest able to participate significantly in protests. Prior to the information revolution, to deal with the collective action problem inherent in mass protests, it was necessary the presence of a strongly organized group willing to incur on prohibitive costs of coordination. That is why protests before the 2000s usually had clear leaderships, like labor unions or political parties, and homogeneous demands, as only individuals sympathetic to these organized groups would participate on the protests. Now, with internet, social medias and mobile phones, both bilateral communication and mass-communication are virtually free, which makes it much cheaper to organize a protest. This explains why recent protests have been marked by horizontality (absence of strong leaderships) and heterogeneous demands, since groups with very different agenda can overcome the collective action problem.Castells (2013)

To understand how this changes on the organizational structure of protest affect the reaction of the governments, we work with a symmetric network with 6 groups. There is one player, who we call the organized group (O), that has a better communication technology, parametrized by c_h, meaning that the participation of the group's members is more known to every individual in society. The other 5 groups, which we call disorganized, have a worse communication technology, parametrized by $c_l \leq c_h$.[3] We model it by assuming that the influence group J has in group K is multiplied by the communication technology $0 < c_J \leq 1$. So, the network is given by

$$G(c_h, c_l) = \begin{pmatrix} c_h & c_l\lambda & \cdots & c_l\lambda \\ c_h\lambda & c_l & \cdots & c_l\lambda \\ \vdots & \vdots & \ddots & \vdots \\ c_h\lambda & c_l\lambda & \cdots & c_l \end{pmatrix} \tag{3-5}$$

In order to understand in term of choice of repression when effective technologies of communication are popularized - like in the recent information revolution - we present the following numerical exercise: first, we start with a matrix $G(c_h, c_l)$ with $c_h = 1$ and $c_l = 0.1$, to represent a society before the information revolution. We interpret such a setting as one in which an organized group was able to invest in an expensive communication technology, while the other groups depend on more rudimentary ways of communication, like word-of-mouth. Then, we increase progressively the parameter c_l, until the technology of communication of all groups is equal to 1, representing a society in which everyone can communicate freely using social media or mobile phones.

We show the results on figure 3. The green line shows the intercentrality of the key group, which invariably will be the organized group. The blue line, shows the net benefit of repression, as given by inequality (3-4), which is the aggregate participation conditional on repression minus the aggregate increase in aggrievement. We plot for such graphs, varying λ, which parametrizes the complementarity of the network $G(c_h, c_l)$.

The first thing to notice is that, in all the cases, the intercentrality of the organized group is much higher than the net benefit of repression. This occurs because their capacity of communication is so much better than that of the other groups that their removal from the network drastically reduces aggregate protest participation. Therefore, the government is much more likely to deal with protests peacefully by giving a concession to the organized group than by violence in the world pre-revolution. On the other hand, as the communication capability of the other groups catch up, this cease to be true as the other groups lose their dependence on the organized group to mobilize a large number

[3]The number of groups is irrelevant.

of protesters. Eventually, the deterrence power of repression becomes a more attractive solution to the government than concession to the key group. This may explain why so many recent protests have been faced with police force.[4].

A second point that can be made from figure 3 is that as the complementarity on the network increases (λ), the faster the government chooses repression over concession. This happens because communication is complementary to the strength of the social ties between groups. Then, for a given communication technology, a society with stronger social ties will have higher aggregate participation, which explains why the key group loses its importance faster when λ increases.

We have shown that the likelihood of repression increases both with improvement in communication technology and strength of social ties, but is this bad news for the protesters? Not necessarily. First of all, even though protests in societies with worse communication technology and weaker social ties are less likely to be repressed, the social movements will lose steam when the organized, key group receives a concession. If the interests of this group is align with that of the rest of society, this may be just fine, but otherwise the protests will only benefit a small segment of society. Second, as either communication technology is improved or social ties become stronger, the incumbent's probability of reelection is reduced. The reason for that is the aggregate protest participation becomes higher with these changes in society independently of the government's choice between concession and repression. We formalize this result on the following proposition.

Proposição 3.5 *If the communication technology is improved (higher c_l) or the social ties become stronger (higher λ) the incumbent's probability of reelection becomes smaller.*

[4]Ortiz et al. (2013), using data from more than 800 protests worldwide, show that roughly 50% of the protests between 2006 and 2013 were faced with some kind of repression - although they cannot differentiate between political repression and possible excesses from law enforcement.

4
Unpopular Groups

In the last section we exemplified our theory using networks of complementarity only. Nevertheless, the results developed above are more general, applying to settings in which groups may cause negative participation externality in each other. In this section, we analyze one such setting, with unpopular groups, whose members' participation discourage the participation of individuals from the remainder groups, which we call the moderate groups.

We start our analysis by focusing in a setting with one unpopular group whose members do not care about the participation of n moderate groups. We allow for a completely general matrix of complementarity between the moderate groups and, for now, we let the negative externality that members from the radical group exert on the moderate groups to be symmetrical, parametrized by the letter h, for hatred.[1] Formally, the network is given by

$$G(h) = \begin{bmatrix} G_{-U} & -h \cdot \mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix} \tag{4-1}$$

where G_{-U} is an arbitrary non-negative NxN matrix, representing the complementarity between the moderate groups, and $\mathbf{1}$ and $\mathbf{0}$ are the n-dimensional vectors of ones and zeros, respectively.

We say that a group is unpopular when $h < 0$ and it is consolidated when its participation in $t = 2$, after learning which is the network of connections G, is large. One way a group may be unconsolidated is for its individual characteristics on $t = 2$, α_2^R, be equal to zero, but as we shall see later in this section, this is not the only way it can happen.

Unless stated otherwise we will consider a general strictly increasing cost function with $C(0) = 0$. Thus, in opposition to the analysis of section 2, it may now be rational for the government to concede to no group even if it decides not to repress the protest. We will call this option "ignoring the protest". The following result shows how the presence of a very unpopular consolidated group may harm a protest.

Proposição 4.1 *Suppose the unpopular group is consolidated ($\alpha_r^U > 0$) and concession is always costly ($C(\mathbf{S}) = 0 \Leftrightarrow \mathbf{S} = \emptyset$). There exists $\bar{h} > 0$ such that*

[1] The assumption that the members of the unpopular group do not care about other group's participation really simplifies the analysis because it makes the participation of the moderate groups an affine function of h.

$h > \bar{h}$ implies that the government ignores the protests. Also, the incumbent's probability of reelection increases with h.

The presence of an unpopular group may harm the protest when its participation deters the other groups' participation strongly enough so that the movement weakens itself over time. From the point of view of removing the incumbent from office, this is bad news, since although repression generates disutility to the protesters, it at least increases the probability of the incumbent losing the election.

It is not certain, though, that an unpopular group will always be consolidated, or even that it exists. On the next three subsections we explore different ways in which the government can consolidate, or even create, an unpopular group in order to weaken the protests.

4.1 Violent Group

On this subsection we assume that the radical group is not aggrieved by redistributive issues (absence of concession). Formally, we suppose that the individual characteristics of the group when there is no repression, α_0^U, is close to zero. Therefore, members of the unpopular group participate very mildly on the first period of protests and they remain out of the streets when there is no repression.

Thus far, we have assumed that repression has the effect of reduce the participation of each group by assuming that the increase in the cost of protesting (v^J) was larger than the increase in aggrievement (δ^J) for every group. In this section we maintain the assumption that $v^m - \delta^m < 0$ for every moderate group, while assuming, on the other hand, that the unpopular group is violence prone, meaning that $v^U - \delta^U > 0$. In other words, the participation of the unpopular group increases with repression. Hence, the government has an additional reason for repressing the protests. As before, it serves as a way to discourage the participation of the moderate groups by increasing their cost in protesting, but now it also consolidates the unpopular group, which further reduces the participation of moderates.

For this analysis to be interesting, it is necessary that the political cost of concession when $h = 0$, given by $\zeta X(G_{-S^*}) + C(\mathbf{S}^*) \equiv \min_{\mathbf{S} \in 2^{\mathbf{N}}} \{\zeta X(G_{-S}) + C(\mathbf{S})\}$, is bigger than $\left(\Delta + \frac{\alpha_1^U}{1-\phi}\right)$, which is the cost of repression when every moderate individual finds it too costly to protest. In other words, the maximum cost of concession in h must be smaller than the minimum repression cost in h.

Proposição 4.2 *Suppose* $\zeta X(G_{-S^*})+C(\mathbf{S}^*) > \Delta+\frac{\alpha_1^U}{1-\phi}$*. If the unpopular group is not consolidated (*$\alpha_0^U < \epsilon$*), there exists* $\tilde{h}$ *such that if* $h > \tilde{h}$ *and there are still some individuals protesting after repression, then the government repress the protest. Also, the incumbent's probability of reelection increases with the unpopularity of the radical group.*

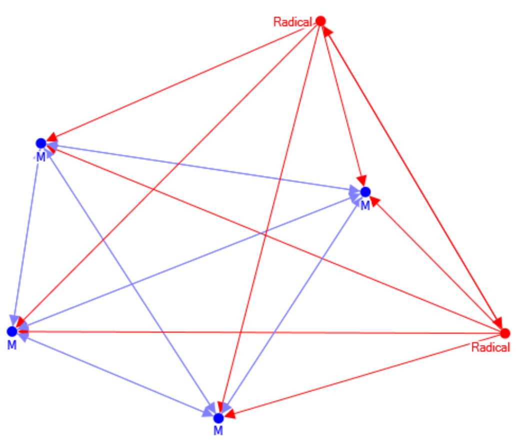
Proposition (4.2) asserts that when the radical group is unconsolidated and unpopular enough, the government represses the protest. The message is that when the deterrence effect of the unpopular group is large enough, the government will find it worthwhile to repress the protest in order to consolidate the presence of that group in $t = 2$. Also, making matters worse for the protesters, not only protests are more likely to be repressed but the incumbent's probability of being ousted of office diminished with the unpopularity of the radical group.

We have seen in this section how the government can use repression as a way to consolidate a group that is both unpopular and violence prone. On the next section we will see, on a slightly different setting, a way in which the government may consolidate an unpopular group through a concession.

Remark We ask for interiority in proposition (4.2) because for any $\alpha^R > 0$ there will be a high enough h such that ignoring the protest is optimal. This is the result of proposition (4.1). There, this result might occur just when all the groups stop protesting. Here, we show that repression is optimal whenever at least one group is still protesting.

4.2
Competing Radical Groups

Figure 4.1: Radical Network



In this subsection we apply the model to understand how the government can use unpopular groups to weaken protests without resorting to violence. We maintain the same structure as in the rest of this section, but now, instead of only one unpopular group, there are two unpopular group. We also restrain the analysis for a symmetrical network of complementarities between the n moderates, parametrized by λ. The negative influence the unpopular groups exert on the moderates is still $-h$ and the unpopular groups exert a negative externality of participation in each other parametrized by $-\gamma$. To further simplify the analysis, we let $\alpha^J = \alpha^M$ if J is moderate and $\alpha^J = \alpha^R$ if J is radical for $\alpha^J, \alpha^R \in \mathbb{R}_+$. Figure 4 summarizes the situation, in which the arrows indicate the direction of the influence, the blue line means positive influence and the red line means negative influence. A nice way of interpreting this setting is that the unpopular groups are radical ones from the opposite sides of the society's ideological spectrum.

Even though both radical groups are indifferent to the presence of moderates, they refuse to knowingly participate with each other in a protest. Thus, if the radicals really despise each other, their participation in $t = 2$ will be very small, allowing a large mass of moderates to protest. In other words, both radical groups, even though unpopular, are not consolidated because of the presence of the other. In order to consolidate one of the radical groups, it may be in the best interest of the government to give a concession to the other radical groups, removing it from the protests. Then, the participation of members from the remaining radical group soars, reducing the participation from the moderates in $t = 2$.

We will not discuss the possibility of repression in this subsection, focusing exclusively on the government's decision of concession, that is far more interesting in this setting. We also assume that the government can give at most one concession, as in the previous section, because it allows us to focus on the question of when a radical group will be the network's key groups.

When there is only one radical, the government will obviously never give it a concession, since its removal always increases aggregate participation. When there are two radical groups though, it also increases the participation of the other radical group which diminishes the participation from the moderates. In the proposition above we present conditions for the intercentrality measure of the two radical groups to be positive, which is equivalent to say that the increase in participation from the radical is offset by the decrease in participation from the moderates.

Proposição 4.3 *Let d_r be the intercentrality of the radical group. The removal* *of a radical group reduces aggregate protest participation ($d_r > 0$) if*

$$\gamma > \frac{1-\phi}{\phi} \tag{4-2}$$

$$h + (n-1)(\lambda + h) > \frac{1-\phi}{\phi} \tag{4-3}$$

Condition (4-2) states that it will be worthwhile to withdraw one radical from the network only if the hatred among the radical groups is big enough. This makes sense, since on the contrary taking one radical off will increase the participation of the other radical too mildly. Then, the moderate groups will appreciate that change since there will be an entire radical group off the protests without a lot more of the other radical protesters.

Condition (4-3) states that the sum of the complementarity between moderates and the aversion of moderates towards the radicals must be large enough in order to withdrawing one radical from the network diminish aggregate participation. The role of h in the condition is straightforward, since the higher the unpopularity of the radical groups, the larger effect its presence has on the moderates' participation. The presence of λ is more interesting, working as a multiplier effect. When λ is high, the presence of a consolidated radical group discourages moderates' participation both directly, from its unpopularity, and indirectly, from less participation of other moderate groups.

Even though it is possible that aggregate participation is reduced by conceding to a radical group, this does not mean that the radical will ever be the key group of the network. It is also possible that the intercentrality of the radical is increasing in some parameter but that the intercentrality of the moderates is increasing even more quickly. Thus, the following proposition analyzes what happen to the difference in intercentralities, $d_r - d_m$, when we increase the magnitude of the parameters defining the network of connections,(λ, h, γ).

Proposição 4.4 *Suppose equations 4-2 and 4-3 hold. The following results hold*

(a) The intercentrality of the radical $d_r(h)$ is increasing in h, while the intercentrality of the moderates $d_m(h)$ is decreasing in h. In the absence of interiority concerns, there is a unique point $h(\gamma, \alpha^R, \lambda))$ such that the two intercentralities cross.

(b) The function $h(\gamma, \alpha^R, \lambda)$ is decreasing in α^R.

(c) If we strengthen condition (4-3) to $h + (n-2)(\lambda + h) > \frac{1-\phi}{\phi}$*, the function* $h(\gamma, \alpha^R, \lambda)$ *is decreasing in* γ*.*

Item (a) of the proposition above states that as the radical groups become more unpopular, eventually it will be worthwhile to concede to them. The reason the intercentrality of both radical groups increase with their unpopularity is that the presence of one of the radical suppresses the participation of the other, a suppression that gets more important to aggregate participation as the negative participation externality from the radical to the moderate groups increase. The result that eventually it will be optimal to concede to the radical group is curious, because it is a group that generates negative externalities to every other group.

The other two items are comparative statics performed on the point $h(\gamma, \alpha^R, \lambda)$, which is the minimum level of unpopularity required for a radical group to be the key group of the network. Item (b) is intuitive: increases in α^R engender higher participation of both radicals, implying that it will be optimal to concede to one of them for lower levels of unpopularity.

The result from item (c), on the other hand, is less straightforward. As γ increases, the participation of both radicals is reduced on the original network. This fosters further participation from the moderates when both radical groups are still on the network. On the other hand, conditional on conceding to one of the radical, the parameter γ has no effect on aggregate participation, because the network without one of the radicals is not affect by it. Then, the option of not conceding to a radical becomes relatively less attractive when γ gets higher. This effect has the counter-intuitive feature that the less participative a radical group is (because of γ), the more likely they are to receive a concession, i.e. they receive a concession for smaller levels of unpopularity. Lastly, when we do not have the strengthen condition the results are inverted. This happens because on that situation the decrease in participation of the radicals when γ increases offsets the decrease in participation of the moderates. This is the same condition as required in proposition 4.3, but for $n-1$ moderates.[2]

So far we have studied ways in which the government, through repression and concession, can consolidate the participation of a radical unpopular group. Nevertheless, not every social movement will be frequented by an unpopular group to begin with. The next subsection addresses the question of how the government can foster hatred towards a group in society.

[2] The comparative statics with the complementarity of the moderate groups sub-network, λ, is ambiguous, depending on the parameters considered. We show that in the appendix.

4.3 Creation of an Unpopular Group

In this section we extend the model by assuming the government may invest in links of hatred between the groups. The idea is the same as in the rest of this section, but now the government can create an unpopular group. The first simplification in our model is that the government can create hatred towards one group only. We name that group as R, for radical. There are other n groups, which are generically called M, for moderates.

The original network of connections is represented by the (N+1)x(N+1) network G given by

$$G = \begin{bmatrix} G_{-R} & \mathbf{w} \\ \mathbf{0} & 1 \end{bmatrix} \tag{4-4}$$

where G_{-R} is an arbitrary NxN matrix, $\mathbf{w} \in \mathbb{R}^n$ is a vector representing the influence group R originally has on the other group. For simplicity, we assume that group R individuals are not affected by the participation other groups' members, since as already discussed, this assumption considerably simplifies the analysis. [3] Although unnecessary for the general formulation of the problem, we also assume that the matrix G_{-R} is of complementarities because that yields sharper and more interesting comparative statics results.

The government may pay a cost $H(p)$, for $p \geq 0$, to substitute the original matrix G for the matrix $G(p)$ given by

$$G(p) = \begin{bmatrix} G_{-R} & \mathbf{w} - p \cdot \mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix} \tag{4-5}$$

where $\mathbf{1}$ is the n-dimensional vector of 1's. Therefore, the government can create a radical group by affecting the negative externality group R has on the other groups. The idea is that the government may spend an amount of money in propaganda against one specific group in order to make members of the other group unhappy with protesting side by side with member from that group. We will abstract of how propaganda affects the choice between concessions and repression in this subsection, focusing only on the optimal choice of propaganda.

Conditional on not conceding, the choice of propaganda p^* is the solution to the following problem

$$p^* \in \min_p \{\zeta X(G(p)) + H(p)\} \tag{4-6}$$

where $X(G(p))$ is the aggregate participation given the network $G(p)$, which means, as shows in section 2, that $X(G(p)) = \mathbf{1}'[\mathbf{I} - \phi G(p)]^{-1}\boldsymbol{\alpha}$. The following

[3] It also does not alter the result qualitatively if we assumed an arbitrary non-positive vector $\mathbf{u}$ instead of $\mathbf{0}$.

lemma characterizes the government's problem.

Lemma 3 *$X(G(p))$ is an affine function in p and the solution is unique for a cost function H such that $H' > 0$ and $H'' > 0$.*

As already stressed, the fact that the aggregate participation is linear on p makes the comparative statics much more straightforward. We state on the proposition below the main comparative statics on the optimal choice of propaganda.

Proposição 4.5 *(i) For any network G_{-R}, the optimal level of propaganda increases with ζ and γ^R and decreases with μ_0.*
(ii) For any non-negative matrix G_{-R}, the optimal level of propaganda increases with ϕ and any entry of the matrix G_{-R}.
(iii) The optimal level of propaganda is not affected by the parameters $\mathbf{w}$, γ_m for each moderate group m.

Item (i) of the proposition states the fact that the government will find it worthwhile to invest more in negative propaganda when the marginal protester hurts its popularity more (ζ). Also, when the radical group participates more, either because it is more aggrieved (γ^R) or because its members' cost of opportunity in protesting is smaller (μ_0), the government will invest more in propaganda, since the return of this investment is proportional to the number of radical participating.

Item (ii) concerns the propagation of unpopularity on the network. When we increase either the diffusion parameter ϕ, meaning that the individuals are more reactive to small changes in their utility, or the network G_{-R} becomes denser, meaning that the complementarities are propagated to more players (or more intensely to some players) on the sub-network of the moderates, the government invest more in propaganda against the radical. Here, the return to investment becomes higher because the propagation of unpopularity becomes more efficient. Once more, the assumption that the moderates' network is of complementarities is crucial to the result as the decrease in participation of every moderate caused by the higher diffusion is

Finally, item (iii) shows that the optimal level of propaganda is not affected by the participation of each moderate (γ^m) and the initial externality the radical group exert on the other groups. This result is an implication of the linearity of the program (lemma 3).

5
Conclusion

We study in this paper a sequential game played between a government and groups of protesters connected by a completely general network of influence. This setting allows us to understand how characteristics of the groups in society affect the pattern of protest participation as well as the likelihood of the choices of concession and repression by the government. We find that more horizontal protests, i.e., movements without strong recognizable leaderships, are more likely to be dealt with repression than protests with strong leaderships. As an application of our result, we studied how the recent communication revolution allowed more horizontal protest which are both more likely to be repressed and more likely to oust the incumbent out of office.

We also explored how a radical unpopular group may harm the protest. We show that as the hatred towards this group increases, the more probable the incumbent's reelection is. We also showed three different ways in which the government can explore radical groups: first, if a radical group is violence-prone, the government may initially repress the protests in order to attract that unpopular group which reduces the participation in protests of moderate groups in society. Second, when the society is populated by two extremist groups from the opposite side of its ideology spectrum, the government may concede to one of the groups in order to stimulate the participation of the other group. If this second group is unpopular with the moderate groups of the society, the increased presence of one extremist group may significantly reduce their aggregate protest participation. Finally, we slightly extended the model to allow the government to make political propaganda to generate hatred in against a particular group, creating thus an unpopular group.

The complexity of the interaction between governments and social movements has led us to abstract from certain aspects of the phenomenon that could be explored in future applications. First, we do not explicitly model how protests and repression affects the popularity of incumbent politicians. This is still a puzzling subject on political economy theory, and competing theories have not yet been extensively tested empirically.[1] A possible way of doing so was to incorporate in our setting a habit formation or a social interaction model. Second, the political competition is very simplified and could become

[1]see Madestam et al. (2013) for an excellent first step on this direction.

much richer, with organized groups, such as lobbies, competing for influence with disorganized groups of protesters. Finally, the network of connections is assumed to be exogenous. An interesting next step is to try to understand how the connections can be formed endogenously by the participants in protests.

Bibliography

ACEMOGLU, D.; ROBINSON, J. A.. **Economic origins of dictatorship and democracy**. Cambridge University Press, 2005.

AGUILLAR, A.; FERRAZ, C.. **Public services quality, social media and collective action**.

AKERLOF, G. A.; KRANTON, R. E.. **Economics and identity**. *Quarterly journal of Economics*, pp 715–753, 2000.

BALLESTER, C.; CALVÓ-ARMENGOL, A. ; ZENOU, Y.. **Who's who in networks. wanted: the key player**. *Econometrica*, 74(5):1403–1417, 2006.

BALLESTER, C.; ZENOU, Y. ; CALVÓ-ARMENGOL, A.. **Delinquent networks**. *Journal of the European Economic Association*, 8(1):34–61, 2010.

BELHAJ, M.; DEROÏAN, F.. **Strategic interaction and aggregate incentives**. *Journal of Mathematical Economics*, 49(3):183–188, 2013.

BERKOWITZ, L.. **Frustrations, comparisons, and other sources of emotion arousal as contributors to social unrest**. *Journal of Social issues*, 28(1):77–91, 1972.

BLATTMAN, C.; MIGUEL, E.. **Civil war**. *Journal of Economic Literature*, pp 3–57, 2010.

BONACICH, P.. **Power and centrality: A family of measures**. *American journal of sociology*, pp 1170–1182, 1987.

BUENO DE MESQUITA, B.; SMITH, A.; SIVERSON, R. M. ; MORROW, J. D.. **The logic of political survival**. Cambridge, ma: mit Press, 2003.

CASTELLS, M.. **Networks of outrage and hope: Social movements in the internet age**. John Wiley & Sons, 2013.

CASTELLS, M.. **Communication power**. Oxford University Press, 2013.

DHILLON, A.; PERALTA, S.. **Economic theories of voter turnout**. *The Economic Journal*, 112(480):F332–F352, 2002.

FEDDERSEN, T. J.. **Rational choice theory and the paradox of not voting**. *Journal of Economic Perspectives*, pp 99–112, 2004.

GATES, S.. **Recruitment and allegiance the microfoundations of rebellion**. *Journal of Conflict Resolution*, 46(1):111–130, 2002.

GERBER, A. S.; HUBER, G. A. ; WASHINGTON, E.. **Party affiliation, partisanship, and political beliefs: A field experiment**. *American Political Science Review*, 104(04):720–744, 2010.

GERBAUDO, P.. **Tweets and the streets: Social media and contemporary activism**. Pluto Press, 2012.

GLAESER, E. L.; SUNSTEIN, C. R.. **A theory of civil disobedience**. Technical report, National Bureau of Economic Research, 2015.

GOULD, D.. **Passionate political processes: bringing emotions back into the study of social movements**. *Rethinking Social Movements: Structure, Meaning and Emotion. Lanham MD: Rowman and Littlefield Publishers*, pp 155–175, 2003.

GRANOVETTER, M.. **Threshold models of collective behavior**. *American journal of sociology*, pp 1420–1443, 1978.

GROSSMAN, G. M.; HELPMAN, E.. **Special interest politics**. MIT press, 2002.

GURR, T. R.. **Why men rebel**. Princeton, Princeton University, 1970.

HAGGARD, S.; KAUFMAN, R. R.. **Inequality and regime change: Democratic transitions and the stability of democratic rule**. *American Political Science Review*, 106(03):495–516, 2012.

JASPER, J. M.. **The art of moral protest: Culture, biography, and creativity in social movements**. University of Chicago Press, 1997.

LINDBECK, A.; WEIBULL, J. W.. **Balanced-budget redistribution as the outcome of political competition**. *Public choice*, 52(3):273–297, 1987.

LOHMANN, S.. **A signaling model of informative and manipulative political action.** *American Political Science Review*, 87(02):319–333, 1993.

LOHMANN, S.. **The dynamics of informational cascades: the monday demonstrations in leipzig, east germany, 1989–91**. *World politics*, 47(01):42–101, 1994.

LOHMANN, S.. **Information aggregation through costly political action**. *The American Economic Review*, pp 518–530, 1994.

MADESTAM, A.; SHOAG, D.; VEUGER, S. ; YANAGIZAWA-DROTT, D.. **Do political protests matter? evidence from the tea party movement***. *The Quarterly Journal of Economics*, pp qjt021, 2013.

MCADAM, D.; TARROW, S. ; TILLY, C.. **Dynamics of contention**, volume 2. Cambridge Univ Press, 2001.

MILGROM, P.; ROBERTS, J.. **Rationalizability, learning, and equilibrium in games with strategic complementarities**. *Econometrica: Journal of the Econometric Society*, pp 1255–1277, 1990.

MISCHE, A.. **De estudantes a cidadãos**. *Revista Brasileira de Educação*, (6), 1997.

MULLAINATHAN, S.; WASHINGTON, E.. **Sticking with your vote: Cognitive dissonance and political attitudes**. *American Economic Journal: Applied Economics*, 1(1):86–111, 2009.

MURPHY, K. M.; SHLEIFER, A.. **Persuasion in politics**. *American Economic Review*, 94(2):435–439, 2004.

MYERSON, R. B.. **Large poisson games**. *Journal of Economic Theory*, 94(1):7–45, 2000.

OLSON, M.. **Logic of collective action: public goods and the theory of groups, the..** *Harvard Economic Studies*, 1965.

ORTIZ, I.; BURKE, S. L.; BERRADA, M. ; CORTÉS, H.. **World protests 2006-2013**. *Initiative for Policy Dialogue and Friedrich-Ebert-Stiftung New York Working Paper*, (2013), 2013.

PASSARELLI, F.; TABELLINI, G.. **Emotions and Political Unrest**. CESifo Working Paper Series 4165, CESifo Group Munich, 2013.

PERSSON, T.; TABELLINI, G.. **Political economics: explaining economic policy**. *Zeuthen lecture book series Show all parts in this series*, 2000.

TERRA, C.; RUEDIGER, T.. **Bresil : retour vers le futur ?** *Politique etrangere*, 4/2013 (winter), 2013.

VAN STEKELENBURG, J.; KLANDERMANS, B.. **The social psychology of protest**. *Current Sociology*, pp 0011392113479314, 2013.

A
Appendix

A.1
Proofs Benchmark Model

Lemma 1

Interiority. Let G be such that $G_{ij} \in [-\kappa, \kappa]$ and $G_{ii} > 1$ $\forall i$ and $\theta^K = \mathbb{E}(a^{iK})$. Also, define $\bar{\mu} = \max_J\{\mu + v^J\}$, $\bar{\theta} = \max_J(\theta^J) = \max_J\{\delta^J + \gamma^J\}$, and $\min_J(\theta^J) = \min_J\{\gamma^J\}$.[1]

Let $a^K = \mu^K - \mathbf{e}'_K G\mathbf{x}$ be the marginal player of group K. As $x_k = Pr(a^{iK} > a^K) = 1 - F(\tilde{a}^k)$, we need to state conditions such that $0 < Pr(a^{iK} > a^K) < 1$. Since a^{iK} is distributed on the interval $[\theta^K - \frac{1}{2\phi}, \theta^K + \frac{1}{2\phi}]$, it is necessary and sufficient for interiority that for every K,

$$\mu^K - \sum_J G_{KJ}[1 - F(a^J)] \in (\bar{a}^K - \frac{1}{2\phi}, \bar{a}^K + \frac{1}{2\phi}).$$

Thus, for the above condition to hold, it is sufficient that the following two conditions are satisfied:

$$\underline{\theta} + \frac{1}{2\phi} > \bar{\mu} + \kappa(N-1) \tag{A.1.1}$$

$$\bar{\theta} - \frac{1}{2\phi} < \mu_0 - \kappa N \tag{A.1.2}$$

We obtain those conditions by substituting for, respectively, the largest and the least possible value of equilibrium participation on a general network. By definition, $\mu_0 \leq \bar{\mu}$, which implies that

$$\underline{\theta} + \frac{1}{2\phi} - \kappa(N-1) \geq \bar{\theta} - \frac{1}{2\phi} + \kappa N \Rightarrow$$
$$\frac{1}{\phi} \geq \kappa(2N-1) + \bar{\theta} - \underline{\theta} \geq \kappa(2N-1)$$

Therefore, $\phi \leq \frac{1}{\kappa(2N-1)}$

Uniqueness. To ensure uniqueness, the matrix $(I - \phi G)$ must be invertible. By Gershgorin circle theorem, a sufficient condition for the

[1] The minimum value of μ^K is μ_0, since all groups share the initial cost of participation μ_0 and $v^K > 0$ $\forall K$.

invertibility of this matrix is

$$(1-\phi G_{ii})-\phi\sum_{j\neq i}|G_{ij}|>0$$

which is implied by

$$1-\phi\kappa N>0\Leftrightarrow\phi<\frac{1}{\kappa N}$$

QED.

Proposition 1

The players' utility is given by $U^{iJ}(\rho,\mathbf{x};a^{iJ},G)=\rho[a^{iJ}-\mu^J+\sum_K G_{JK}x_K]$, where ρ is 1 if the agent protest and zero otherwise and $a^{iJ}=\delta^J r+\gamma^J(1-c^J)(1-r)+\epsilon^{iJ}$. The choice variable is trivially supermodular and if the matrix G is non-negative, the variables ρ and x^K for every K are increasing-differences (ID). Therefore, the game is supermodular. It is straightforward to see that the parameters γ^J and δ^J are ID with ρ and the parameters μ_0 and v^J are decreasing differences with ρ, which means that the game is indexed by $(\gamma^J,\delta^J,-\mu_0,-v^J)$. The comparative statics results follow from the Theorem 6 in Milgrom e Roberts (1990).

The fact that an increase in any of the entries of G increase aggregate participation is quite intuitive and follows from Corollary 1 of Belhaj e Deroïan (2013).

Equilibrium aggregate participation in given by $\mathbf{1}'\sum_{k=0}^{\infty}\phi^k(G^k\boldsymbol{\alpha}(\phi))$. For every k, the number $\mathbf{1}'(G^k\boldsymbol{\alpha})$ is non-negative, implying that the derivative of $\mathbf{1}'\sum_{k=0}^{\infty}\phi^k(G^k\boldsymbol{\alpha})$ in ϕ is positive. Also, $\alpha_J=\frac{1}{2}+\phi(\theta^J-\mu^J)$, which is increasing in ϕ, which implies that $\mathbf{x}(G)$ is increasing in ϕ.

Finally, the network G is strongly connected if and only if there exist a natural m such that $G^m_{KL}>0\;\forall K,L$. Thus, an increase in α^L increases $G^m_{KL}*\alpha^L$ without decreasing any other quantity. Also, suppose there is an increase on the entry G_{KL}. There is a directed path from K to any other node J which implies that part of the participation of J is given by the product of the weight of the connections on the path, multiplied by ϕ^{l+1}, where l is the length of that path. Thus, on the entry G_{KL} also increases this term which increases the participation of group J. Note that it is possible that $J=K$, but it is trivial since all the nodes in G are assumed to have a self-loop.

Lemma 2

Let $D^J=D^J(Q^I;Q^O)=W^J(Q^I)-W^J(Q^O)$ and $\theta^J(S,r)=\delta^J r+\gamma^J(1-c^J)(1-r)$ the average aggrievement of group J individuals. The share of votes the incumbent receives from group J is given by

$$\pi^J = \frac{1}{2} + \phi\left[D^J + \xi - \theta^J(S,r)\right] \tag{A.1.3}$$

The total share of votes received by the incumbent is given by $\Pi^I = \frac{1}{N}\sum_{J=1}^{N}\pi^J$, which is given by

$$\Pi^I = \frac{1}{2} + \frac{\phi}{N}\left\{\sum_J\left[D^J + \xi\right]\right\} - \frac{\phi}{N}\sum_J\theta^J$$

The probability of the incumbent being reelected, $Pr(I)$ is given by

$$\begin{aligned} Pr\left(\Pi^I \geq \frac{1}{2}\right) &= Pr\left(\xi + \frac{1}{N}\sum_J D^J - \frac{1}{N}\sum_J\theta^J > 0\right) \Leftrightarrow \\ &= Pr\left(\xi > \frac{1}{N}\sum_J\theta^J - \frac{1}{N}\sum_J D^J\right) \end{aligned}$$

Substituting for ξ this expression yields

$$Pr\left(\tilde{\xi} > \frac{1}{N}\sum_J\theta^J - \frac{1}{N}\sum_J D^K + f(r,\mathbf{S},\mathbf{x}^1,\mathbf{x}^2) + C(\mathbf{S}) - \bar{y}\right) \Leftrightarrow$$

We have that

$$Pr(I) = 1 - H\left(\frac{1}{N}\sum_J\theta^J - \frac{1}{N}\sum_J D^K + f(r,\mathbf{S},\mathbf{x}^1,\mathbf{x}^2) + C(\mathbf{S}) - \bar{y}\right)$$

Finally, by the symmetry of H we have

$$Pr(I) = H\left(\frac{1}{N}\sum_J D^J - \frac{1}{N}\sum_J\theta^J - f(r,\mathbf{S},\mathbf{x}^1,\mathbf{x}^2) - C(\mathbf{S}) + \bar{y}\right)$$

On the electoral period, the only term of the above expression which is not a sunk cost is $\sum_J D^J = \frac{1}{N}\sum_J\left[W^J(Q^I) - W^J(Q^O)\right]$. As H is strictly increasing, the problem of the incumbent is to maximize the function $\frac{1}{N}\sum_J W^J(Q^I)$. The problem of the opponent is symmetrical to the incumbent's problem, and if the solution is unique, which is the case as W is concave, it is straightforward that the optimal platform announcement is the same to the incumbent and the opponent.

Thus, substituting $\theta^J(S,r)$ for $\delta^J r + \gamma^J(1-c^J)(1-r)$, the probability of reelection in equilibrium is given by:

$$H\left(-\frac{1}{N}\sum_J(\delta^J r + \gamma^J(1-c^J)(1-r)) - f(r,\mathbf{S},\mathbf{x}^1,\mathbf{x}^2) - C(\mathbf{S}) + \bar{y}\right)$$

We can incorporate the term $-\frac{1}{N}\sum_{J\notin S}\bar{a}^J$ on the cost function with no loss of generality, implying that the incumbent's probability of reelection in equilibrium is

$$Pr(I) = H\left(-r\frac{1}{N}\sum_{J}\delta^J + \bar{y} - f(r, \mathbf{S}, \mathbf{x}^1, \mathbf{x}^2) - C(\mathbf{S})\right).$$

QED.

Proposition 2

From the utility of the citizens in protesting, it is straightforward to see that the choice of participation is increasing-differences with $(a^{ij} - \mu^J)$. As the utility of protesting is trivially supermodular on own choice of protesting, and, since G is of complementarities, increasing-differences on other group's participation, the participation of each group increases when $(a^{ij} - \mu^J)$. Now, saying that $\delta^J > v^J$ for every J implies that $(a^{ij} - \mu^J)$ given repression in larger than $(a^{ij} - \mu^J)$ with no repression, implying that aggregate participation increases with repression. Finally, this implies that

$$\zeta\sum_{J\in\mathbf{N}} x_J^2(G, r=1) > \zeta\sum_{J\in\mathbf{N}} x_J^2(G, r=0) \geq \min_{\mathbf{S}\in 2^{\mathbf{N}}}\left\{\zeta\sum_{J\notin\mathbf{S}} x_J^2(G_{-S}, r=0) + C(\mathbf{S})\right\}$$

The last inequality from assuming that $C(\emptyset) = 0$. QED

Proposition 4 Let $X(G, r)$ be the aggregate protest participation. By definition

$$d_{J^*} = X(G, r=0) - X(G_{-J^*}, r=0) \qquad \text{(A.1.4)}$$

The government will repress iff

$$\zeta[X(G_{-J^*}, 0)] > \Delta + \zeta X(G, 1) \Leftrightarrow$$
$$\zeta[X(G_{-J^*}, 0) - X(G, 1)] > \Delta \Leftrightarrow$$
$$\zeta[X(G, r=0) - d_{J^*} - X(G, 1)] > \Delta$$

A simple rearrangement yields

$$d_{J^*} < X(G, 0) - X(G, 1) - \frac{\Delta}{\zeta} \qquad \text{(A.1.5)}$$

QED.

Proposition 5

Item (i) comes from the fact that

$$X(G, 0) - X(G, 1) = \phi\mathbf{1}'(\mathbf{I} - \phi G)^{-1}\cdot(\mathbf{v} - \boldsymbol{\delta})$$

which implies that

$$d_{J^*} < \phi \mathbf{1}'(\mathbf{I} - \phi G)^{-1} \cdot (\mathbf{v} - \boldsymbol{\delta}) - \frac{\Delta}{\zeta} \tag{A.1.6}$$

Both vectors $\mathbf{v}$ and $\boldsymbol{\delta}$ only appear on the RHS of the equation and, for a non-negative matrix G, it is straightforward to that an increase in any of the components of $\mathbf{v}$ or a decrease in any of the components of $\boldsymbol{\delta}$ (which also decreases $\Delta = \sum \delta^J$) increases the likelihood of repression.

Finally, (iii) comes naturally from the original government's tradeoff $\zeta[X(G_{-J^*}, 0)] > X(\Lambda, 0) + \zeta X(G, 1)$. The only term above affected by an increase of intensity on the links connected to the key group is $X(G, 1)$. When G is a non-negative matrix, it is easy to see, by the traditional strategic complementarity arguments, that an increase of intensity in any tie increases aggregate participation, $X(G, 1)$. Therefore, an increase in any of the key group's ties reduces the likelihood of repression.

Proposition 6

The probability of being ousted of office, after a monotone transformation, is given by $L(r, \mathbf{S}) = r\Delta + \zeta \mathbf{1}' X(G_{-S}(r; c_l, \lambda)) + C(\mathbf{S})$, where $\mathbf{1}' X(G_{-S}(r; c_l, \lambda))$ is the aggregate participation. From proposition 1, we know that aggregate protest participation increases with c_l and λ, for any G_{-S} and r, because this is always a network of complementarities. Then, the government's problem is increasing. Hence, applying the envelope theorem for arbitrary sets shows that the derivative of the government's value function is increasing with c_l and λ, which implies that the probability of being ousted of office increases with these parameters. QED

A.2 Proofs Unpopular and Violent Groups

Proposition 7

The loss function (A.5.3) goes to zero as h goes to infinity when the government chooses $\mathbf{S} = \emptyset$ and $r = 0$, since eventually every group stop protesting (all of the groups have corner solutions). The same does not happen when $\mathbf{S} \neq \emptyset$ or $r \neq 0$, since both actions have fixed costs (i.e., independent of how many individuals protest on $t = 2$.). Thus, when h goes to infinity this functions flatten in a positive constant.

Also, a trivial application of the envelope theorem for arbitrary sets shows that the derivative of the value function with h is negative, meaning that the loss function is strictly decreasing on h. Thus, the probability of reelection increases with unpopularity of the radical group, h.

QED.

Proposition 8 (Violent Groups)

The total cost of repression is given by[2]

$$R(h) \equiv \zeta \mathbf{1}' \left(M_{11}\boldsymbol{\alpha}_1^M - h \cdot \frac{\alpha_1^R}{1-\phi} M_{11}\mathbf{1} \right) + \Delta \tag{A.2.1}$$

while the total cost of concession is given by

$$C(h) \equiv \zeta \mathbf{1}' \left(M_{11}^{-i^*}\boldsymbol{\alpha}_0^{M^{-i^*}} - h \cdot \frac{\alpha_0^R}{1-\phi} M_{11}^{-i^*}\mathbf{1} \right) + c \tag{A.2.2}$$

If there was no problem of interiority, the solution of the problem would follow from the fact that both functions are lineear on h and the absolute value of repression loss function is larger in absolute value than the concession loss function, since $|R'(h)| > |C'(h)|$ iff

$$\alpha_1^R \mathbf{1}' M_{11}\mathbf{1} > \alpha_0^R \mathbf{1}' M_{11}^{-i^*}\mathbf{1}$$

Both $M_{11}\mathbf{1}$ and $M_{11}^{-i^*}\mathbf{1}$ are vector of equilibrium participation with non-negative network which by proposition (2.3) implies that the aggregate participation $\mathbf{1}'M_{11}\mathbf{1}$ is larger than $\mathbf{1}'M_{11}^{-i^*}\mathbf{1}$, since $M_{11}^{-i^*} \leq M_{11}$ component-wise. Finally, we assumed that group R is violence prone, which implies that $\alpha_0^R < \alpha_1^R$. So the result is proven if there is no interiority concern.

The fact that the incumbent's probability of reelection increases with h follows the same logic as proposition 4.1. QED

For a more general result, which does not depend on the interiority condition, we present the following demonstration. Let $\underline{h}$ be such that $\mathbf{x}(G(h), r = 1) = 0 \Leftrightarrow h > \underline{h}$. This is the unpopularity point in which all groups besides the unpopular cease to protest when there is repression.

First we prove that for a sufficiently small $\alpha_0^R, \mathbf{x}(G_{-i^*}(h), r = 0), \mathbf{x}(G(h), r = 0) > 0$ component-wise for every $0 \leq h \leq \underline{h}$.

Dem. It suffices to show that it is valid for $h = \underline{h}$, since the functions are decreasing in h. Let $\underline{\alpha} = \min_k \{\boldsymbol{\alpha}_0^{M^{-i^*}}, \boldsymbol{\alpha}_0^M\}$ If $\alpha_0^R < \frac{1-\phi}{\underline{h}} \cdot \underline{\alpha}$, then every participation is interior (look at equation **??** bellow). The same logic applies for $\mathbf{x}(G_{-i^*}(h), r = 0)$. Then, the functions C and I, the later the loss function of ignoring the protest, are linear for $0 < h < \underline{h}$, while the function $R(h)$ is a piecewise linear convex function that becomes flat from $\underline{h}$ on. We have assumed that both $C(0) = \zeta X(G_{i^*}) + c$ and $I(0) = \zeta X(G)$ are larger than $R(\underline{h}) = \Delta) + \frac{\alpha_1^U}{1-\phi}$.

Since C and I are linear functions, we just need to make sure that α_0^R is small enough so both $C(\underline{h}) = C(0) - \underline{h}|C'(h)| > R(\underline{h})$ and $I(\underline{h}) = I(0) - \underline{h}|I'(h)| > R(\underline{h})$. We can do that since $|C'(h)|$ and $|I'(h)|$ are continuous

[2]See the notation and demonstration at Additional Proofs below.

on α_0^R and zero for $\alpha_0^R = 0$. There are two cases, and we just prove with the function C because the other prove is analogous.

(i) If $C(0) > R(0)$ then we simply make $|C'(h)|$ small enough so $C(\underline{h}) > R(0)$.

(ii) If $R(0) > C(0)$ then because $C(\underline{h}) > R(\underline{h})$ and continuity, the functions must cross at least once before $\underline{h}$. Suppose they cross again before $h = \underline{h}$. Then, they must cross a third time for the same reason as they did at least once. But this can only happen if there are point $h', h'' \in (0, \underline{h})$ such that $|R'(0)| > |C'|$, $|R'(h')| < |C'|$ and $|R'(h'')| > |C'|$. Since C' is constant this violates the convexity of R, implying that the curves cross once and only once. Note that the function R is differentiable a.e.

A.3
Proofs Competing Radical Groups

Proposition 9 The intercentrality of the radical and the moderate group when there are n moderates, respectively, is given by

$$d_r(n) = \frac{2\alpha_r}{1-\phi+\gamma\phi} - \frac{\alpha_r}{1-\phi} +$$

$$n\left[\frac{\alpha_m(1+\gamma\phi-\phi)-2h\phi\alpha_r}{(1-(n-1)\lambda\phi-\phi)(1-\phi+\gamma\phi)} - \frac{\alpha_m(1-\phi)-h\phi\alpha_r}{(1-(n-1)\lambda\phi-\phi)(1-\phi)}\right]$$

$$d_m(n) = n\left[\frac{\alpha_m(1+\gamma\phi-\phi)-2h\phi\alpha_r}{(1-(n-1)\lambda\phi-\phi)(1-\phi+\gamma\phi)}\right] -$$

$$(n-1)\left[\frac{\alpha_m(1+\gamma\phi-\phi)-2h\phi\alpha_r}{(1-(n-2)\lambda\phi-\phi)(1-\phi+\gamma\phi)}\right]$$

Thus,

$$d_r(n) = \frac{2\alpha_r}{1-\phi+\gamma\phi} - \frac{\alpha_r}{1-\phi} + \frac{nh\phi\alpha_r}{1-(n-1)\lambda\phi-\phi}\cdot\left(\frac{1}{1-\phi} - \frac{2}{1-\phi+\phi\gamma}\right) =$$

$$= \alpha_r\left(\frac{1}{1-\phi} - \frac{2}{1-\phi+\phi\gamma}\right)\left(\frac{nh\phi}{1-(n-1)\lambda\phi-\phi} - 1\right) =$$

$$= \alpha_r\left(\frac{1}{1-\phi} - \frac{2}{1-\phi+\phi\gamma}\right)\left(\frac{\phi[1+nh+(n-1)\lambda]-1}{1-(n-1)\lambda\phi-\phi}\right)$$

Finally, we have

$$d_r(n) = \frac{\alpha_r \phi}{(1-(n-1)\lambda\phi - \phi)(1-\phi+\phi\gamma)}.$$

$$\left(\frac{\phi}{1-\phi}\gamma - 1\right)\left(1 + nh + (n-1)\lambda - \frac{1}{\phi}\right)$$

Therefore, a sufficient condition for $d_r(n) > 0$ is that the two following conditions hold:

$$\gamma > \frac{1-\phi}{\phi}$$
$$h + (n-1)(h+\lambda) > \frac{1-\phi}{\phi}$$

QED

Proposition 10

$$d_m(n) = \left(\alpha_m - \frac{2h\phi\alpha_r}{1-\phi+\gamma\phi}\right) \cdot \left[\frac{n}{1-(n-1)\lambda\phi-\phi} - \frac{n-1}{1-(n-2)\lambda\phi-\phi}\right] \tag{A.3.1}$$

where the first expression is positive because of interiority and the second is always positive. It is easy to see that the expression is linear and decreasing in h. The expression for the intercentrality of the radical in 19 makes it clear that it is a linear increasing function in h. When inequality 4-3 is binding the radical's intercentrality is zero and the intercentrality of the moderates is always positive, implying that the functions cross once and only once, at the point $h(\gamma, \alpha^R, \lambda))$.

Item (b) comes from the implicit function theorem (IFT). Since h is defined implicitly by $F(h;\gamma,\alpha^R,\lambda) \equiv d_r(h;\gamma,\alpha^R,\lambda) - d_m(h;\gamma,\alpha^R,\lambda) = 0$ which, as we have seen, has a positive derivative in h, the signal of any partial derivative of $h(\gamma,\alpha^R,\lambda)$ is the negative of the respective partial derivative on $F(h;\gamma,\alpha^R,\lambda)$. It is easy to see that that $\frac{\partial d_r(h;\gamma,\alpha^R,\lambda)}{\partial \alpha^R}$ is positive while $\frac{\partial d_m(h;\gamma,\alpha^R,\lambda)}{\partial \alpha^R}$ is negative, implying that $\frac{\partial F(h;\gamma,\alpha^R,\lambda)}{\partial \alpha^R} > 0$.

Item (c) also uses the IFT:

Note first that $d_r - d_m = X(G) - X(G_{-r}) - (X(G) - X(G_{-m})) = X(G_{-m}) - X(G_{-r})$. Taking into account only terms that depend on γ, this expression becomes

$$d_r - d_m \overset{\gamma}{=} \frac{2\alpha_r}{1-\phi+\gamma\phi} - h\phi\alpha_r \frac{2(n-1)}{(1-\phi+\gamma\phi)[1-(n-2)\lambda\phi-\phi]}$$
$$= \frac{2\alpha_r}{1-\phi+\gamma\phi}\left(1 - \frac{h\phi(n-1)}{1-(n-2)\lambda\phi-\phi}\right)$$

The term $\frac{2\alpha_r}{1-\phi+\gamma\phi}$ is decreasing on γ, which implies that the sign of the derivative on γ of $d_r - d_m$ will be positive iff $\left(1 - \frac{h\phi(n-1)}{1-(n-2)\lambda\phi-\phi}\right) < 0$, or equivalently

$$h(n-1) + \lambda(n-2) > \frac{1-\phi}{\phi} \tag{A.3.2}$$

Finally, we show that an increase in λ has an ambiguous effect. Deriving $d_r - d_m$ with respect to λ yields

$$\frac{(n-1)(n-2)\phi}{[1-(n-2)\lambda\phi-\phi]^2}\left(\alpha_m - \frac{2h\phi\alpha_r}{1-\phi+\gamma\phi}\right) - \frac{n(n-1)\phi}{[1-(n-1)\lambda\phi-\phi]^2}\left(\alpha_m - \frac{h\phi\alpha_r}{1-\phi}\right)$$

We have that $\frac{n(n-1)\phi}{[1-(n-1)\lambda\phi-\phi]^2} > \frac{(n-1)(n-2)\phi}{[1-(n-2)\lambda\phi-\phi]^2}$ and $\alpha_m - \frac{2h\phi\alpha_r}{1-\phi+\gamma\phi} > \alpha_m - \frac{h\phi\alpha_r}{1-\phi}$, which makes that difference dependent on the magnitude of the parameters. The last inequality comes from the assumption that $\gamma > \frac{1-\phi}{\phi}$. QED.

A.4 Proofs Propaganda

Lemma 3 The proof of the linearity of aggregate participation follows lemma A.1 below. When H is increasing and convex, there is only one solution because either $H'(0)$ is smaller than the slope of $\mathbf{1}'X^M(p)$, and it crosses this value from bellow and only once or $H'(0)$ is larger and the solution is $p^* = 0$. QED

Proposition 11 The government's program is equivalent to

$$\min_p\{[-p \cdot \frac{\zeta\alpha^R}{1-\phi}\mathbf{1}'M_{11}\mathbf{1} + H(p)\} \tag{A.4.1}$$

Since it is a minimization problem and p is trivially supermodular, the monotone comparative statics results can be applied by analyzing if p and the other variables are decreasing-differences (DD).

The quadratic form $\mathbf{1}'M_{11}\mathbf{1}$ is always positive because of the interiority conditions. Thus, $(p,\zeta),(p,\alpha^r)$ are DD. Since $\mathbf{1}'M_{11}\mathbf{1}$ is just the sum of the elements of M_{11}, the cross derivative of the objective function with p and any entry of M_{11} is negative, which implies that they are DD.

Finally, we have seen that participation increases when ϕ increases and G is positive, so the cross derivative of ϕ and p is also negative, implying that (ϕ, p) are DD.

A.5 Additional Proofs

Lemma A1. Suppose the network of connections is given by $G(h) = \begin{bmatrix} G_{-R} & \mathbf{w} - h \cdot \mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}$ where $\mathbf{w}$ is an arbitrary n-dimensional vector. The aggregate participation, given any choice of repression (r) or concession $(\mathbf{S})$ is affine on h.

Dem. Let P be the matrix $\mathbb{I} - \phi G(h)$ and $M = P^{-1}$. Then, we have

$$P = \begin{bmatrix} P_{11} & -\phi(\mathbf{w} - h \cdot \mathbf{1}) \\ \mathbf{0} & 1 - \phi \end{bmatrix} \tag{A.5.1}$$

where P_{11} is a nxn matrix. P is a block triangular matrix and the interiority of the equilibrium under the matrix $G(0)$ guarantees that P_{11} is non-singular. Thus, the inverse of P is given by

$$M = \begin{bmatrix} P_{11}^{-1} & \frac{1}{1-\phi} P_{11}^{-1}(\mathbf{w} - h \cdot \mathbf{1}) \\ \mathbf{0} & \frac{1}{1-\phi} \end{bmatrix} \tag{A.5.2}$$

The vector of participation is $\mathbf{x} = M\boldsymbol{\alpha}$. Define $M_{11} = P_{11}^{-1}$ and partition $\boldsymbol{\alpha} \equiv (\boldsymbol{\alpha}^M, \alpha^R)$. Then, the vector of participation from the n moderates is given by

$$\mathbf{x}^M = M_{11}\boldsymbol{\alpha}^M + \frac{\alpha^R}{1-\phi} M_{11}(\mathbf{w} - h \cdot \mathbf{1}) = M_{11}\boldsymbol{\alpha}^M + \frac{\alpha^R}{1-\phi} M_{11}\mathbf{w} - h \cdot \frac{\alpha^R}{1-\phi} M_{11}\mathbf{1}$$

and the participation of the radical is $\frac{\alpha^R}{1-\phi}$. From equation (**??**), it is clear that the participation vector $\mathbf{x}^M(h)$ is an affine vector function of h. Thus, the government's problem is minimizing the loss function

$$L(r, \mathbf{S}) = r \sum_J \delta^J + \zeta \mathbf{1}' \mathbf{x}^M (G_{-S}) + C(\mathbf{S}) + \frac{\alpha_r^R}{1-\phi} \tag{A.5.3}$$

where $\mathbf{x}^M(N - S)$ is the moderates vector of participation supposing that the moderate groups on $\mathbf{S}$ have received concessions, which is also affine on h for choices of S and r. QED