Pontifícia Universidade Católica
do Rio de Janeiro

Fernando Castro de Campos Roriz

Essays in Financial Economics

Tese de Doutorado

Thesis presented to the Postgraduate Program in Economics from the Departamento de Economia, PUC-Rio as a partial fulfillment of the requirements for the degree of Doutor em Economia.

Advisor: Prof. Marcelo C. Medeiros

Rio de Janeiro

September 2014

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Bibliographic Data

Roriz, Fernando

Essays in Financial Economics / Fernando Castro de Campos Roriz; advisor: Marcelo C. Medeiros – 2014

76 f. : il.; 30 cm

Tese (Doutorado em Economia) - Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2014.

Inclui referências bibliográficas.

1. Economia – Teses. 2. Economia Financeira; Incerteza Política; Partidarismo; Precificação de Ativos; Eleições. I. Medeiros, Marcelo. II. Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia. IV. Título.

Acknowledgements

Gostaria de agradecer a todos aqueles, lembrados e não lembrados, que passaram pelo meu caminho em todos estes anos de formação acadêmica.

Agradeço primeiramente à minha família pelo suporte imensurável desde a minha saída de Goiânia. Aos meus pais, Marcelo e Graça, por sempre acreditarem em todos os meus objetivos, muitas vezes mais do que eu. Aos meus irmãos e melhores amigos, Marcelo e Guilherme, que apesar da distância sempre me acompanharam de perto pelas minhas andanças. Aos meus avós, e em especial ao meu avô materno Adélio que, na sua simplicidade, nos ensinou que o conhecimento era o maior bem de uma pessoa. E a todos os meus outros familiares que, de longe ou de perto, sempre estiveram muito presentes.

Agradeço também a todos do departamento de economia da PUC-Rio pelos anos de convivência. Um agradecimento especial ao meu orientador, Marcelo Medeiros, pelo estímulo e otimismo sem fim em relação aos seus alunos e orientandos, fazendo-nos sempre chegar a um ponto acadêmico além do que achávamos capazes.

Agradeço também aos membros da banca avaliadora pelos relevantes comentários e cuidados ao avaliar a tese.

Um agradecimento especial a todos os meus amigos desde os tempos de colégio. Aos velhos amigos do Ateneu e Visão, agradeço ao Fernando, Pedro, Vinicius, Rafael, Fabrício, Jabber, Luisa e Núbia pela longa amizade. Aos amigos do IBMEC-SP, agradeço ao Luiz e ao Caio, pelas minhas primeiras discussões como economista e companheirismo nos anos de São Paulo. Aos amigos da PUC-Rio, agradeço ao Arthur, Adão, Breno, Augusto, Juan e Rapha, pelos anos inesquecíveis no Rio de Janeiro.

Por último, agradeço o apoio financeiro da CAPES, do CNPq pelos auxílios concedidos, sem os quais este trabalho não poderia ter sido realizado.

Abstract

Roriz, Fernando; Medeiros, Marcelo (advisor). **Essays in Financial Economics.** Rio de Janeiro, 2014. 76p. Tese de Doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This dissertation is composed of two articles in financial economics. The first article tests for the effects of political uncertainty on the stock market. Based on an index of political uncertainty constructed from the winning probability of U.S. presidential candidates calculated in the months prior to the 2004 Presidential Elections, we find that an increase in this index tend to depress considerably more stocks with higher exposure to the market factor, even after controlling for partisanship effect and overall market volatility. Thus, it seems to be the case that when investors face an increase in political uncertainty they prefer stocks with smaller exposure to the market risk factor, once these assets increase (or decrease less) their hedging ability in periods of high uncertainty. In the absence of a state contingent market to fully diversify electoral risk, these findings show that consumers could respond to the wealth uncertainty generated by the electoral process holding stocks less exposed to the market factor. In the second article, I develop a model in which elections move asset prices. The model allows for the party in power to have an impact over the profitability of firms, interpreted as the partisanship effect. This feature makes stock prices respond to the uncertainty caused by the electoral race and the upcoming election. Under this source of uncertainty, investors start requiring a higher risk premium to hold assets subject to the election result. Also, it is shown that in general, regardless of which party is best for firms' profitability, investors demand a risk premium to hold their stock shares during election. Finally, we discuss some results of the literature from the model's perspective. The first article is co-authored with Marcelo Medeiros.

Keywords

Partisanship; Electoral Uncertainty; Stock Prices; Elections.

Resumo

Roriz, Fernando; Medeiros, Marcelo (orientador). **Essays in Financial Economics.** Rio de Janeiro, 2014. 76p. Tese de Doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Esta dissertação é composta de dois artigos em economia financeira. O primeiro artigo analisa os efeitos da incerteza política no mercado de ações. Usando um índice de incerteza política construída a partir das probabilidades de vitória dos candidatos presidenciais dos EUA calculados nos meses antes das eleições de 2004, temos que um aumento neste índice tende a deprimir consideravelmente mais ativos com maior exposição ao risco de mercado, mesmo após controlarmos pelo efeito de partidarismo e volatilidade global do mercado. Assim, parece ser o caso que os investidores quando enfrentam um aumento da incerteza política preferem ações com menor exposição ao fator de risco de mercado, uma vez que esses ativos aumentam a sua proteção em períodos de elevada incerteza. Dado a falta de um mercado contingente onde as pessoas podem diversificar o risco eleitoral, estes resultados mostram que os consumidores podem responder à incerteza gerada pelo processo eleitoral através de ações menos expostas ao risco de mercado. No segundo artigo, desenvolve-se um modelo em que as eleições afetam os preços dos ativos. O modelo permite que o partido no poder para tenha um impacto sobre a lucratividade das firmas, interpretado como o efeito de partidarismo. Esta característica faz com que os preços das ações respondam à incerteza gerada pela corrida eleitoral e pela eleição. Neste contexto, investidores avessos ao risco exigem um prêmio maior para manter em sua carteira ativos sujeitos ao resultado da eleição. Além disso, mostra-se que, em geral, independente do partido que é melhor para a lucratividade das empresas, os investidores exigem um prêmio de risco para carregar esses ativos durante a eleição. Por último, discutimos alguns resultados da literatura utilizando as previsões do modelo. O primeiro artigo é coautorado com Marcelo Medeiros.

Palavras-chave

Partidarismo; Incerteza Eleitoral; Preços de Ativos; Eleições.

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1 Political Uncertainty and Equity Risk Premium: Evidence from the 2004 U.S. Presidential Election

1.1.Introduction

One day after the first debate between US presidential candidates in the 2004 election, George Bush's winning probability was 65.9%. From October 1st to 14th, one day after the last debate, this probability fell to 54.5%. Around one week later, Bush's winning probability peaked at 62%, but at 4 p.m. in the Election Day this number was down to 52%. As a matter of fact, exit polls released around 3 p.m. predicted a Bush defeat, which was gradually reviewed as votes were counted. Despite the great effort of analysts to predict the outcome, political uncertainty was a central theme in this election.[1] This paper tests whether there is a premium associated with political uncertainty that can explain the cross-section of expected returns.

Risk averse investors facing time of high uncertainty should protect themselves by reducing the correlation of their portfolio with market movements. Inversely, these same investors should require higher expected returns for holding assets that covary strongly with the market. These stocks would command a higher risk premium because they reduce investors' hedging ability in periods of high uncertainty. The central hypothesis here is that fluctuations in political uncertainty generate priced risk factors that can explain the cross-sectional variation in stock returns.

Using evidence from the period preceding the 2004 U.S. Presidential Election, this paper attempts to disentangle political effects from overall market movements. Working with prediction market data, we first develop an index of political uncertainty based on the winning probability of U.S. presidential candidates calculated in the months prior to the election. Then, using stock returns from the S&P 500 components, we find that an increase in this index tend to depress considerably more stocks with higher exposure to the market factor, even

[1] See, for example, 'Uncertainty Reigns as Nov. 2 Nears' in the New York Times, October 31, 2004.

after controlling for partisanship effect and overall market volatility. Thus, as expected, it seems that, when investors face an increase in political uncertainty, they prefer stocks with smaller exposure to the market risk factor, once these assets increase (or decrease less) their hedging ability in period of high uncertainty. In the absence of a state-contingent market to fully diversify the electoral risk, these findings show that consumers could respond to the wealth uncertainty generated by the electoral process holding stocks less exposed to the market factor.

Endogeneity problems caused by the simultaneous effects between the stock market and political uncertainty could be a potential harm to our empirical strategy. We try to address this problem by (1) working with data close to the elections, when the economic conditions are fairly stable and the variability in political uncertainty is mainly driven by the upcoming election, (2) through the use of a great span of assets, once a single firm is unlikely to have a significant impact in the political uncertainty index, and (3) using control variables that help to disentangle the overall market movements from the political uncertainty movements that we are interested in.

The remainder of this paper is organized as follows. In the next section, we review the related literature. Section 3 presents a brief background about the 2004 US presidential election and the data. In Section 4, we develop the political uncertainty index and relate to other uncertainty measures during the period prior to the election. In Section 5, we present the methodology. Section 6 contains the empirical results. In Section 7, we run a series of robustness tests to validate our findings. Finally, Section 8 concludes the paper.

1.2.Related Literature

Uncertainty and its effects on macro variables has been a major topic in the economic literature and relies on the argument that since many investments are irreversible, agents would prefer to delay their decisions and wait for more information to take them (Bernanke, 1983; Romer, 1990; Pindyck, 1991). Uncertainty causes significant impact in stock markets (Zhang, 2006; Ozoguz, 2009) and, more specifically, political uncertainty can be one of the reasons for agents to delay their actions.

The literature has widely related political events to stock market movements. Bittlingmayer (1998) uses German data from 1880 to 1940 to show that political uncertainty generated by the transition from Imperial to Weimar Germany was a relevant source of volatility, which had negative effects on output. Santa Clara and Valkanov (2003) look historically at partisanship effects in the U.S.. Using data from 1927 to 1998, they document that average excess return in the stock market is higher under Democratic presidencies than under Republican presidencies.

Another strand of the literature works with cross-section data, cross-country or within-country. Durnev (2010) studies how political uncertainty surrounding national elections affects investment-to-price sensitivity. Using a sample of 79 countries, he finds that corporate investment is 40% less sensitive to stock prices during election years than during non-election years, probably due to stock prices becoming less informative. In a similar work, Julio and Yook (2012) use firm-level data for 48 countries and find that corporate investment falls by an average of nearly 5 percent in the year leading up to a national election relative to other years. Belo, Gala and Li (2013) find that during Democratic presidencies, firms with high government exposure experience higher cash flows and stock returns. On the other side, the opposite holds true during Republican presidencies, pointing out a relevant channel through which the partisanship effect could work.

A closer literature related literature discusses the partisanship effects under some specific events. Also working with prediction market tracking election outcome and financial data, Snowberg, Wolfers and Zitzewitz (2007) find that Bush's re-election in 2004 led to modest increases in equity prices, nominal and real interest rates, oil prices and the U.S. dollar. The same applies to the 2000 election. This work also finds strong evidence that partisanship, rather than incumbency, is the relevant channel through which equities are affected. Despite the use of the same prediction market data, we look at a different question. Snowberg, Wolfers and Zitzewitz (2007) test if there is a significant effect caused by the election of different parties on aggregated measures. Now, the idea here is to test if the electoral process, regardless of who wins, has a significant impact in the cross-section of returns because the asset is more or less exposed to the market factor.

In the same line, Knight (2007) uses a sample of 70 firms during the 2000 U.S. Presidential Election and creates a 'Pro-Bush' and 'Pro-Gore' portfolio, i.e., firms that would probably perform better under a Bush or Gore presidency, respectively. He shows that platforms are relevant to explain the different performances, increasing the value of the 'Pro-Bush' portfolio as Bush's winning probability went up.[2]

Pastor and Veronesi (2012, 2013) develop a general equilibrium model where investors cannot fully anticipate which policy the government is going to choose. This source of uncertainty generates a series of predictions, including a negative effect of political uncertainty on asset prices because it is not fully diversifiable.

Thus, while there is a large literature relating politics to stock markets, little empirical work has been done on the impact of political uncertainty in the cross-section of expected returns. This paper aims to contribute to this strand of the literature. The closest work is Mattozzi (2005). He creates 'presidential portfolios' using campaign contributions under a Bush versus a Gore presidency, close to what is done in Knight (2007). Given the different portfolio performances, he argues that agents could explore this fact to protect themselves against this source of uncertainty. Our paper differs in both focus and methodology. He focus on the fact that partisanship effects could serve as a tool to hedge against political uncertainty. Here we focus on the fact that political uncertainty is an inherent feature in the electoral process and, despite the partisanship effect, it alone can cause cross-sectional variation in the stock market. Finally, methodologically, our identification comes from the cross-section of returns, while Mattozzi (2005) relies his trading strategy on a time-series analysis, using different performances through time to propose a hedging strategy.

1.3. Data

1.3.1.Prediction Markets

The first data source provides information on the winning probabilities of the Democratic and Republican candidates in the 2004 U.S. Presidential Election.

[2] Knight (2007) uses data from Iowa Electronic Market (IEM), which tracks the probability that each candidate would win the plurality of the popular vote.

During this election cycle, Intrade.com, a prediction market, had a contract that would pay $10 if the Republican candidate George W. Bush were reelected president, and zero otherwise, and an analogous contract for John Kerry, the Democratic candidate. Therefore, the price of these securities can be interpreted as the market estimated probabilities that Bush or Kerry would win the upcoming election.[3] We have obtained intraday data, which allows us to match the market estimated probability to the stock market closing time (We use the last price before 4 p.m (EST)).

Figure 1.1 presents the evolution for the probability of the Republican candidate to win using Intrade.com data throughout Sep 29 until the Election Day (Nov 2) for the 2004 Election. During this period, three debates were held on Sep 30 (D1), Oct 8 (D2) and Oct 13 (D3). George Bush started October ahead of John Kerry, with a winning probability around 66%. One day after the last debate, on Oct 14, Bush's lead had diminished considerably to 54.5%. Around one week later, Bush recovered his lead, reaching a 62% winning probability. But, in the last few days, this advantage almost disappeared, reaching the bottom level of 51% on Oct 29. Finally, even in the Election Day the result seemed very hard to predict, with Bush still holding the lead, but with a narrow margin of only 4 percentage points.

As pointed out by Knight (2007), prediction market data are preferred to tracking poll data for several reasons. First, tracking poll data provide expected vote shares, instead of probabilities of victory. Second, market implied probability is more efficient to predict the outcome of the election.

One advantage about the data used in this paper compared to Knight (2007) is that he used data from Iowa Electronic Market (IEM), which only tracks the probability that each candidate would win a plurality of the popular vote. Although this data is probably a good proxy for the winning probability of each candidate in the upcoming election, it is not a clean measure, since in the U.S. the winner of the plurality of votes is not necessarily the election winner. This happened in the 2000 Presidential Election, when Al Gore won the plurality of votes but George W. Bush was the winner in the Electoral College. Another advantage is that for the period considered by Knight (2007), the median day

[3] See Wolfers and Zitzewitz (2006).

during the sample witnessed just 229 trades in the Bush contract. Here, I use data closer to the election, and during the sample period, the average quantity of trades is 16,152 per day, indicating a liquid market.

1.3.2. Equity Returns

We employ stock market data from the S&P 500 constituents from Jan 1, 2003 to Nov 2, 2004, obtained from Bloomberg. In particular, we use the composition from Sep 30, 2004 and exclude any stock with incomplete data in the sample period.[4] As a proxy to the overall market, we use the S&P 500 index, whereas the risk-free rate is the 30-day T-bill rate taken from Kenneth French's website[5].

Further, we also work with other three sources of data: (1) the CBOE volatility index (VIX), which measures market expectation of near term volatility conveyed by stock index option prices, extracted from the Federal Reserve Bank of Saint Louis database; (2) the Economic Policy Uncertainty Index and the Equity Uncertainty Index, indexes of search results from large newspapers in the U.S., which serves as a proxy for overall economic policy uncertainty and market uncertainty[6]; and (3) the Fama-French factors HML and SMB also taken from Kenneth French's website[7].

1.4. Political Uncertainty Index

We consider two main features to construct the political uncertainty index. First, the index should capture the fact that the uncertainty is higher when the probability of winning of each candidate is closer to 50%. Second, it probably exists a nonlinear increment in political uncertainty. For example, a variation from 10 points to 9 points advantage by one candidate must cause a different effect on political uncertainty when compared to a variation from 2 points to only 1 point advantage.

[4] Seven firms do not have complete data for the time range used in the paper and are dropped from the database. The list of firms used is available upon request.

[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

[6] See Baker, Bloom and Davis (2012) for further details and www.policyuncertainty.com for the data.

[7] See Fama and French (1993).

Bearing that in mind, we introduce the first feature through very simple math using the absolute advantage between the candidates. Next, to guarantee the second property, we calculate the probability density function for the absolute advantage considering an exponential distribution. This ensures that, whenever the difference between the candidates gets closer to zero, the political uncertainty index gets higher at an increasing rate.

So, the political uncertainty index at time t is given by:

$$PUI_t = \lambda . e^{(-\lambda . adv_t)} \quad (1.1)$$

where adv_t is the absolute advantage of the leading candidate at time t and λ is the rate parameter given by $1/\overline{adv}$, where $\overline{adv}$ is the average of the absolute advantage[8].

Figure 1.2 shows the political uncertainty index given by equation (1) from Sep 29 and Nov 2 and the CBOE Volatility Index (VIX), which works as a proxy to overall market volatility. The interesting point here is that such a simple index as the one developed here is able to fairly track the VIX, revealing that market volatility during this period could be closely related to the election process. In the next sections, we try to take into account the possible bias caused by reverse causality in the estimation.

1.5. Methodology

The regression methodology is based on a two-step procedure. We first regress excess returns on the S&P 500 index to capture exposures to market risk:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + u_{it} \quad (1.2)$$

where r_{it} is the daily rate of return on stock i, r_{mt} is the daily return on the S&P 500 index, and r_{ft} is the return on the 30-day T-bill. Equation (1.2) is regressed using data from Jan 1, 2003 to Aug 31, 2004. As in Knight (2007), we then calculate abnormal returns ($\tilde{r}_{it}$), which are net of market returns for the pre-election period, i.e., Sep 30, 2004 through Nov 2, 2004, as follows:

$$\tilde{r}_{it} = r_{it} - [\hat{\alpha}_i + \hat{\beta}_i(r_{mt} - r_{ft})] \quad (1.3)$$

In the second step, using the abnormal returns calculated for each stock in equation (1.3), the estimated coefficient for each asset i, $\hat{\beta}_i$, is now interacted with

[8] We set $\lambda = 1/\overline{adv}$, which is the rate parameter for an exponential distribution. For the sample considered, we have $\lambda = 5.89$. Other values around 5.89 do no change the results significantly.

increments in the political uncertainty index, ΔPUI_t, and a vector of controls X_t that varies over time but not cross-sectionally. We include as control, for example, the VIX index and other relevant independent variables. Then, the second step ia a panel data regression given by:

$$\tilde{r}_{it} = \sum_{j=0}^{p} \gamma_j \left(\hat{\beta}_i \Delta PUI_{t-j}\right) + \sum_{k=0}^{p} \delta_k' \left(\hat{\beta}_i X_{t-k}\right) + \alpha_i + \theta_t + \varepsilon_{it} \quad (1.4)$$

where the first term in parenthesis is the interaction between the market betas and increments in PUI_t and its lags. The second term if the market beta interacted with the set of controls and its lags, α_i is a series of fixed effects parameters and allows form firm-specific trends during the sample period and θ_t represents a vector of time dummies, one for each day.

The key parameter in equation (1.4) are the γ_j's. They allow us to test if there is cross-section variability in response to political uncertainty shocks. It is expected that stocks with higher market exposure (i.e., higher betas) depress considerably more in face of an uncertainty shock when compared to stocks with lower exposure. Then, the coefficients γ_j's are expected to be negative.

1.6. Empirical Results

Using returns on the S&P 500 components, equation (1.2) is estimated between Jan 1, 2003 and Aug 31, 2004, a window that allows a reliable estimation of the market betas. Table 1.1 presents the summary statistics for the first set of regressions. Row (1) summarizes the estimated coefficients related to the market factor. The average coefficient associated to the market return is around one, as expected, once the proxy for the overall market return is the aggregated index. The relevant fact is that there is variability in firms' exposure to the market factor, going from 0.11 to 2.50, which allows different firms to respond asymmetrically when a common market shock occurs. In an extreme case, if all betas where the same, the regression given by equation (1.4) would not make sense and the time dummies would capture all the variation, since the interaction between the estimated betas and increments in the political uncertainty index (*PUI*) would vary only across time, but not cross-sectionally. Finally, the R-squared and the adjusted R-squared, rows (2) and (3), show that the data variability is fairly explained on average (around 30%).

In the second step the data range goes from the day of the first debate until the Election Day. The period considered is intentionally small, close to one month, and corresponds to 24 trading days. This is an important feature for the identification procedure for two reasons: the small window before election is probably when most important voting discussions and decisions are established; and it restricts the time range when shocks other than political could occur and invalidating our identification. This reverse causality problem is the main reason why we do not use the 2008 US Presidential Election in our analysis. The subprime crisis probably drove the market movements during this period, and the election result was probably highly affected by the crisis. Thus, if we try to estimate the effect of political uncertainty into the stock market, it is very likely that the result could be misleaded by the fact that the market affected the election result. For the 2004 election, we could not identify major market events that could clearly affect that election.

Further, to help in the identification process, firm dummies and time dummies are an essential feature. They capture all the bias that could be generated by portfolio specific trends or time specific events. Another important property that helps in the identification is the use of 493 different stocks, once it is unlikely that a single firm would have a significant impact in the political uncertainty index. Finally, as pointed out by the literature (Knight, 2007; Snowberg, Wolfers and Zitzewitz, 2007), partisanship is relevant. In the line of these papers, we control by the increments in the probability of a Bush victory ($\Delta Pr(Bush)$).

Table 1.2 shows the results of the second step considering a number of different specifications. As shown in column 1 and 2 of Table 1.2, the coefficients for the interaction between the estimated betas and increments in the political uncertainty index indicates that portfolios with higher betas suffer significantly more when there is a positive shock in the political uncertainty index. This is an intuitive result because risk averse investors tend to protect themselves through assets that protect them from market movements when they face uncertainty shocks. Further, column 2 also addresses the potential problem of delays in political news reaching prediction markets including as regressors the interactions at time $t-1$. The lag interaction is significant, but Figure 2 points out that this fact could be related to the close link between the evolution of the political uncertainty index and the VIX index.

Thus, one possible source of bias in the estimation is that the political uncertainty index may be proxying for other sources of shocks that could affect differently the cross-section of assets. Everything that affects equally the firms is controlled by the time dummies, and specific firm trends during the sample period are controlled by the firm dummies. But, omitted shocks that are correlated with political uncertainty are a potential harm. Then, equation (1.4) is redefined including as regressors the interaction between the estimated betas and the increments in the CBOE Volatility Index (VIX) at time t and $t-1$, where the VIX serves as a proxy for overall market uncertainty.

We also take into account the interaction between the estimated betas and increments in the probability of a Bush victory. The results including also this interaction are presented in columns (3) and (4) and remain fairly close to columns (1) and (2).

As shown in column 6 of Table 2, the results demonstrate that political uncertainty matters for the cross-section of stock returns, even after controlling for partisanship effect and overall market volatility. The significant coefficient associated with the VIX interactions show that higher market uncertainty tend to depress more assets with higher exposure to the market factor, an identical interpretation to the previous case when we take into account the political uncertainty. Thus, it seems to be the case that these are two distinct effects.

Due to the nonlinearity of the political uncertainty index, a feature that comes from the exponential term in equation (1.1), its quantitative interpretation depends on the specific leading candidate advantage. To exemplify, from Nov 1, 2004 to Nov 2, 2004, George Bush advantage fell from 10.2% to only 4%. This corresponds to an increase in the political uncertainty index equal to 1.425. Considering only the significant coefficient at time t in column 6 for the interaction with the political uncertainty index, an asset with a market exposure equal to the average market beta would face an expected drop in its value due to the political uncertainty increase around $\hat{\gamma}(\bar{\beta} \times 1.425) = -0.86\%$. On the other hand, an asset with an associated beta one standard deviation below the mean, which means a beta equal to 0.636, would face a decrease in value of 0.52%, depreciating about 34 basis point less than the average beta asset, while the asset with an associated beta one standard deviation above the mean would face a decrease in value of 1.21%.

1.7. Robustness

1.7.1. Economic Policy Uncertainty and Equity Market Uncertainty Indexes

In this section, we use as control other two measures that capture broader uncertainty movements that can be a source of bias in the previous estimation. Developed by Baker, Bloom and Davis (2012), the first index, denoted by Economic Policy Uncertainty index (EPU), is constructed using newspapers' articles that contain at least one term from each of 3 sets of terms. The first set is 'economic' or 'economy'. The second is 'uncertain' or 'uncertainty'. The third set is 'legislation' or 'deficit' or 'regulation' or 'congress' or 'federal reserve' or 'white house'. And, to deal with changing volumes of news articles for a given paper over time, they normalize the raw counts of Economic Policy Uncertainty related articles. The second index, denoted by Equity Market Uncertainty index (EMU), is also constructed using newspapers' articles, but in this case is considered articles the term 'uncertainty' or 'uncertain', the terms 'economic' or 'economy' and one or more of the following terms: 'equity market', 'equity price', 'stock market', or 'stock price'. Again, the authors normalize the index to account for the increasing number of newspapers.[9]

In Table 1.3, we include as regressors the EPU and EMU indexes at $t-1$ and t. After we control for increments in the EPU and EMU indexes, the results associated with the baseline coefficients do not change significantly. And the significant coefficients associated with the EPU and EMU indexes are usually negative and go in the same direction as the coefficients associated with the political uncertainty index and the VIX.

1.7.2. Fama-French Factors and Momentum

A very important strand in the finance literature points out the relevance of other risk factors in the cross-section of expected returns, besides the market factor. Here, we consider the other two risk factors, HML and SMB, developed in Fama and French (1993) as sources of cross-section variability of returns. These two factors are calculated using the 100 portfolio returns formed size and book-to-market developed by Fama and French. The first factor, SMB, is associated with

[9] See Baker, Bloom and Davis (2012) for details.

firms' size (value) and it is calcutated as the return of the smallest portfolio minus the return of biggest portfolio. The second factor, HML, is associated with the firms' book-to-market ratio and it is given by the return on the portfolio with the highest book-to-market minus the return on the portfolio with the lowest book-to-market ratio. We also include the momentum factor, MOM, presented in Carhart (1997). The MOM can be calculated by subtracting the equal weighted average of the highest performing firms from the equal weighed average of the lowest performing firms, lagged one month. It tries to capture the tendency for stock prices to continue rising if they are going up and to continue declining if they are going down.

With the three new risk factors, the first and second steps presented earlier are modified. The first step regression is then given by:

$$r_{it} - r_{ft} = \alpha_i + \beta_{MKT}(r_{mt} - r_{ft}) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{MOM}MOM_t + u_{it} \qquad (1.5)$$

Now, instead of calculating the abnormal returns using only the exposure to the market risk factor, the residual of the first step regression gives us the abnormal return associated with the other two Fama and French factors. Then the abnormal return from Sep 30, 2004 to Nov 2, 2004 is now given by:

$$\tilde{r}_{it} = r_{it} - [\hat{\alpha}_i + \hat{\beta}_{MKT}(r_{mt} - r_{ft}) + \hat{\beta}_{SMB}SMB_t + \hat{\beta}_{HML}HML_t + \hat{\beta}_{MOM}MOM_t] \quad (1.6)$$

In the second step, using the abnormal returns calculated for each stock from equation (1.6), the panel data regression is given by:

$$\tilde{r}_{it} = \sum_{j=0}^{p} \gamma_{MKT_j} \left(\hat{\beta}_{MKT_i} \Delta PUI_{t-j}\right) + \sum_{j=0}^{p} \gamma_{SMB_j} \left(\hat{\beta}_{SMB_i} \Delta PUI_{t-j}\right) + \sum_{j=0}^{p} \gamma_{HML_j} \left(\hat{\beta}_{HML_i} \Delta PUI_{t-j}\right) + \sum_{j=0}^{p} \gamma_{MOM_j} \left(\hat{\beta}_{MOM_i} \Delta PUI_{t-j}\right) + controls_{it} + \alpha_i + \theta_t + \varepsilon_{it} \quad (1.7)$$

where $controls_{it}$ represent the set of controls interacted with the estimated betas. The coefficients of interest are the ones associated with the interaction between the estimated betas and the increments in the political uncertainty index. We do not have a prior to the signal associated to HML, SMB and MOM as we have for the coefficient MKT , which we expect to be negative.

Table 1.4 presents the estimation of equation (1.7) associated with a series of controls. Column (1) uses as control only the interaction between estimated betas and increments in the political uncertainty index. As we can see, the result is a little bit different when compared to column (1) in Table 1.2, since now the market betas are jointly estimated with the other two Fama-French factors. But, in the same line, we find that an increase in the political uncertainty index tend to depress considerably more stocks with higher exposure to the market. As we add controls from column (1) to (6), the results get stronger and significant. The other coefficients are significant, specially the one related to size, which is positive, indicating that stocks more exposed to the size factor tend to suffer less when there is a political uncertainty shock. Further, the coefficient related to HML, related to stocks' book-to-market ratio, indicate that firms less exposed to this specific factor tend to suffer less when they face a political uncertainty shock. Finally, the momentum factor indicates that political shocks tend to exacerbate the average returns of high beta firms, i.e., a political uncertainty shock tends to boost the returns for high beta stocks.

1.7.3. Different Window for Beta Estimation

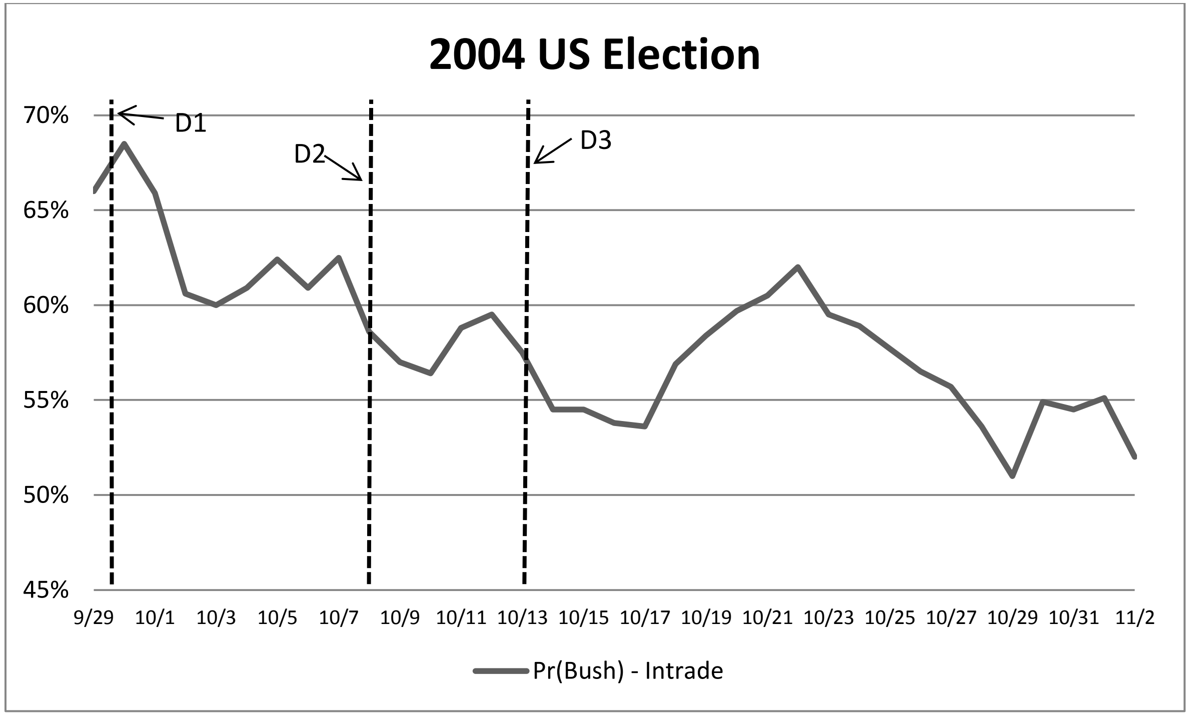
One concern about the first step could be the period used in the regression to estimate the stocks' exposure to the market factor. In the previous case, stock market data went from Jan 1, 2003 to Aug 31, 2004. In table 1.5, we replicate the analysis for a window in the first step ranging from May 1, 2003 to Apr 30, 2004. This is the same window one year before the election considered by Knight (2007).

As we can see, the results still hold, but get statistically weaker in terms of significance. But, again, the results suggest that the cross-section of firms respond heterogeneously to political uncertainty shocks.

1.8. Conclusion

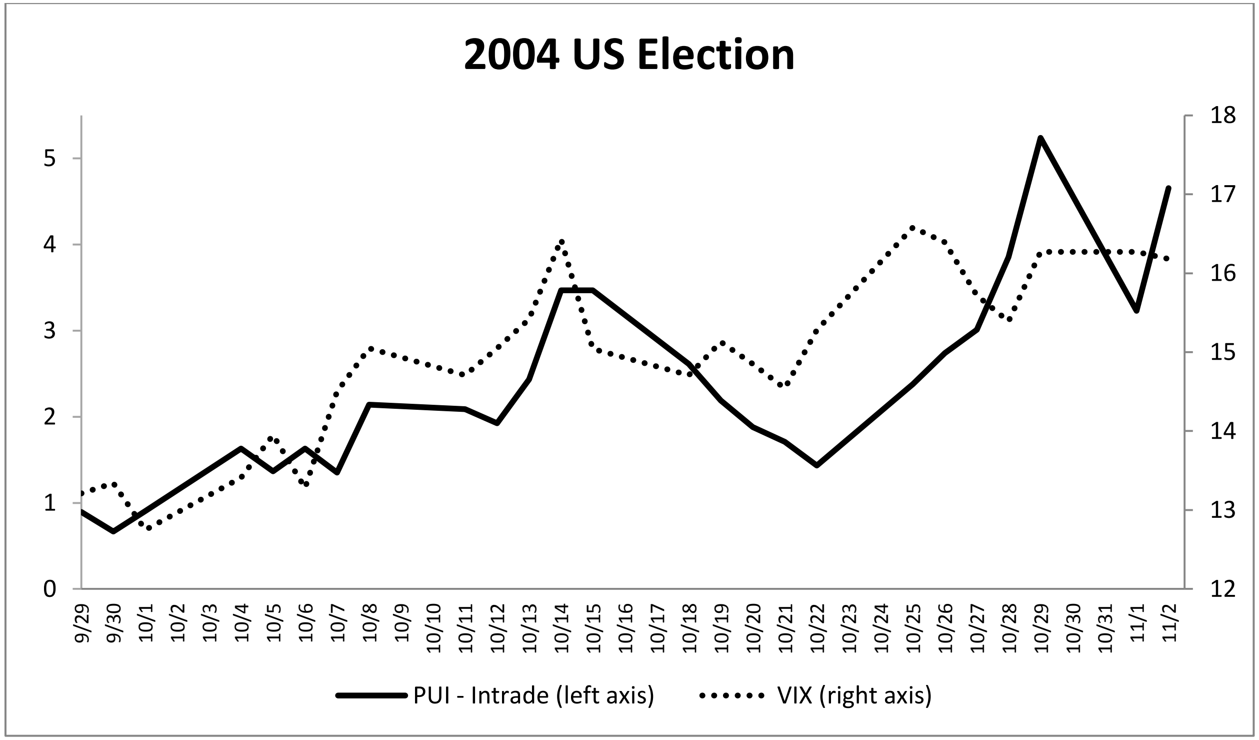
Using evidence from the 2004 US Presidential Election, this paper shows that stocks highly exposed to the market tend to suffer more during times of high political uncertainty. This fact is found even after we control for other variables that our political uncertainty index could be proxying. While many results in the literature have shown the importance of partisanship and political uncertainty over stock market indexes, here we show the relevance of the uncertainty feature in the cross-section of returns, pointing out that the unknown result about the upcoming election may be reflected in equity prices during the electoral process.

Figure 1.1 – Electoral probabilities from Intrade.com – Republican candidate



Notes: This figure shows the evolution of the winning probability for the Republican candidate in the 2004 US Presidential Election from the Intrade.com prediction market.

Figure 1.2 – Political Uncertainty Index and VIX Volatility Index



Notes: This figure shows the evolution of the political uncertainty index and the VIX Volatility Index before the 2004 US Presidential Election.

Table 1.1 - Descriptive Statistics
Summary statistics of the regressions of excess stock returns on the market factor

	Variable	Mean	Std. Dev	Min	Max	Obs.
		2004 Election				
(1)	*beta*	1.053	0.417	0.115	2.500	493
(2)	*R sq.*	0.307	0.133	0.003	0.676	493
(3)	*Adj. R sq.*	0.305	0.134	0.000	0.675	493

Table 1.2 - Baseline Results - 2004 Election
Time-Series Regression - Sample: Jan 1, 2003 - Aug 31, 2004
Panel Regression - Sample: Sep 30, 2004 - Nov 2, 2004

	Intrade Data					
	(1)	(2)	(3)	(4)	(5)	(6)
Beta(i)*ΔPUI(t)	-0.109	-0.256***	-0.197	-0.394**	-0.207	-0.576***
	(0.087)	(0.092)	(0.139)	(0.162)	(0.141)	(0.170)
Beta(i)*ΔPUI(t-1)		-0.488***		-0.454***		-0.293**
		(0.065)		(0.128)		(0.132)
Beta(i)*ΔPr(Bush)(t)			-0.031	-0.049	-0.031	-0.141**
			(0.057)	(0.061)	(0.058)	(0.064)
Beta(i)*ΔPr(Bush)(t-1)				0.013		0.022
				(0.039)		(0.040)
Beta(i)*Δln(VIX)(t)					0.007	-0.004
					(0.012)	(0.013)
Beta(i)*Δln(VIX)(t-1)						-0.078***
						(0.014)
Firm Fixed Effects	yes	yes	yes	yes	yes	yes
Time Fixed Effects	yes	yes	yes	yes	yes	yes
Number of days	24	24	24	24	24	24
Number of stocks	493	493	493	493	493	493
Obs	11832	11832	11832	11832	11832	11832
R-squared	0.013	0.019	0.013	0.019	0.013	0.025

Robust standard erros clustered by firm in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 1.3 - Robustness - EPU and EMU Indexes
Time-Series Regression - Sample: Jan 1, 2003 - Aug 31, 2004
Panel Regression - Sample: Sep 30, 2004 - Nov 2, 2004

	(1)	(2)	(3)
Beta(i)*ΔPUI(t)	-0.116	-0.270*	-0.499***
	(0.101)	(0.157)	(0.163)
Beta(i)*ΔPUI(t-1)	-0.421***	-0.372***	-0.308**
	(0.068)	(0.143)	(0.145)
Beta(i)*ΔPr(Bush)(t)		-0.055	-0.165***
		(0.062)	(0.062)
Beta(i)*ΔPr(Bush)(t-1)		0.022	0.0234
		(0.043)	(0.044)
Beta(i)*Δln(VIX)(t)			-0.0188
			(0.015)
Beta(i)*Δln(VIX)(t-1)			-0.072***
			(0.017)
Beta(i)*Δln(EPU)(t)	0.319**	0.313**	0.318**
	(0.139)	(0.137)	(0.138)
Beta(i)*Δln(EPU)(t-1)	-0.744***	-0.728***	-0.450**
	(0.160)	(0.161)	(0.178)
Beta(i)*Δln(EMU)(t)	-0.305***	-0.295***	-0.07
	(0.065)	(0.064)	(0.077)
Beta(i)*Δln(EMU)(t-1)	-0.294***	-0.308***	-0.174**
	(0.058)	(0.062)	(0.077)
Firm Fixed Effects	yes	yes	yes
Time Fixed Effects	yes	yes	yes
Number of days	24	24	24
Number of stocks	493	493	493
Obs	11832	11832	11832
R-squared	0.025	0.025	0.027

Robust standard erros clustered by firm in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 1.4 - Robustness - Fama and French Factors and Momentum
Time-Series Regression - Sample: Jan 1, 2003 - Aug 31, 2004
Panel Regression - Sample: Sep 30, 2004 - Nov 2, 2004

	(1)	(2)	(3)	(4)	(5)	(6)
Beta_MKT(i)*ΔPUI(t)	-0.065	-0.046	0.264	-0.040	0.130	-0.107
	(0.103)	(0.122)	(0.176)	(0.194)	(0.179)	(0.207)
Beta_SMB(i)*ΔPUI(t)		0.015		0.596***		0.515***
		(0.063)		(0.178)		(0.179)
Beta_HML(i)*ΔPUI(t)		0.100**		0.334***		0.366***
		(0.045)		(0.120)		(0.123)
Beta_MOM(i)*ΔPUI(t)		0.131		1.051***		1.004**
		(0.149)		(0.367)		(0.390)
Beta_MKT(i)*ΔPUI(t-1)	-0.327***	-0.428***	-0.302**	-0.494***	-0.102	-0.377**
	(0.066)	(0.071)	(0.132)	(0.190)	(0.138)	(0.190)
Beta_SMB(i)*ΔPUI(t-1)		0.085		0.415***		0.507***
		(0.070)		(0.137)		(0.146)
Beta_HML(i)*ΔPUI(t-1)		0.069		0.196*		0.188
		(0.043)		(0.114)		(0.116)
Beta_MOM(i)*ΔPUI(t-1)		-0.065		0.803**		0.827***
		(0.126)		(0.311)		(0.314)
Controls						
Partisanship	no	no	yes	yes	yes	yes
VIX	no	no	no	no	yes	yes
Firm and Time Fixed Effe	yes	yes	yes	yes	yes	yes
Number of days	24	24	24	24	24	24
Number of stocks	493	493	493	493	493	493
Obs	11832	11832	11832	11832	11832	11832
R-squared	0.013	0.015	0.014	0.016	0.018	0.026

Robust standard erros clustered by firm in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 1.5 - Robustness - Different Window for Beta Estimation
Time-Series Regression - Sample: May 1, 2003 - Apr 30, 2004
Panel Regression - Sample: Sep 30, 2004 - Nov 2, 2004

	Intrade Data					
	(1)	(2)	(3)	(4)	(5)	(6)
Beta(i)*ΔPUI(t)	-0.118*	-0.254***	-0.093	-0.228	-0.119	-0.396**
	(0.067)	(0.072)	(0.137)	(0.165)	(0.137)	(0.171)
Beta(i)*ΔPUI(t-1)		-0.452***		-0.313**		-0.143
		(0.062)		(0.125)		(0.131)
Beta(i)*ΔPr(Bush)(t)			0.009	0.00405	0.008	-0.0747
			(0.051)	(0.057)	(0.051)	(0.061)
Beta(i)*ΔPr(Bush)(t-1)				0.045		0.062*
				(0.036)		(0.037)
Beta(i)*Δln(VIX)(t)					0.0185*	0.010
					(0.011)	(0.012)
Beta(i)*Δln(VIX)(t-1)						-0.069***
						(0.013)
Firm and Time Fixed Effects	yes	yes	yes	yes	yes	yes
Number of days	24	24	24	24	24	24
Number of stocks	493	493	493	493	493	493
Obs	11832	11832	11832	11832	11832	11832
R-squared	0.013	0.020	0.013	0.020	0.014	0.026

Robust standard erros clustered by firm in parentheses; *** p<0.01, ** p<0.05, * p<0.1

2 Partisanship, Electoral Uncertainty and Stock Prices

2.1. Introduction

All over the world, elections are closely followed by financial markets and asset prices seem to respond significantly to the evolution of political processes. Election results may influence corporate performance by inducing different government policies in areas such as spending and taxing. And, despite the possibility of a good election outcome in the eyes of the market, election induced uncertainty can depress asset prices by itself. In order to best predict these effects, market participants pay attention to a series of signs about who is going to win. For example, party platforms are carefully evaluated during campaigns, debates are watched by millions of voters and newspapers are full of polls, analysis and forecasts about the upcoming election. The objective of this paper is to develop a theoretical model that presents some relevant channels through which partisanship may affect the stock market during the electoral race.

I build on the framework of Pastor and Veronesi (2012, 2013), where asset prices respond to changes in government policy and political news. Their focus is on the fact that investors learn through time about the government effect on firms' profitability. In the present work, I adapt and reinterpret their environment to analyze the behavior of asset prices during the electoral race. I assume a different learning structure in which investors estimate the likelihood of each party winning the election, allowing asset prices to respond to a set of shocks that changes investors' perspective about the election. There is no doubt about the costs and benefits of changing the prevailing party and uncertainty comes from the fact that investors do not know who is going to win.

In my model, there are two parties running for the next election, the incumbent party and the challenger party. At a given point in time, election occurs and its outcome affects firms' profitability, generating different expected utility for agents maximizing their welfare. Investors learn about the election by observing a series of signals coming from the 'political game', which allows them to calculate the winning probability for each party.

There are two sources of uncertainty. First, agents do not know who is going to win the election, which is a by-product of the electoral process. Agents then require a risk premium to hold stocks during the electoral race. This first source of uncertainty is interpreted as the *electoral uncertainty*. The second source of uncertainty is generated by the markets not knowing if the winner party will be able to fulfill all his promises during its term on office. As an example, in the US, it helps if the Congress and the House of Representatives are controlled by the same party as the president. If this is not true, negotiations to approve many of the proposals can be quite difficult. Thus, not knowing the exact partisanship effect over firms' profitability makes investors require higher expected returns to hold assets that are susceptible to this source of uncertainty. This is interpreted as the *partisanship uncertainty*.

The election result establishes which party will be in power for the next term, affecting firms' expected profitability. But, at the same time, it possibly brings a greater amount of uncertainty, increasing discount rates for risk averse investors. So there may be a case where the election result increases the expected cash flow of firms, but has a large enough effect on discount rates that asset prices may actually fall. In some sense, voters choose the mean-variance trade-off from the incumbent party or from the challenger party. Assuming for example that the challenger has higher *partisanship uncertainty*, its effect over firms' profitability must be high enough to guarantee that the cash flow effect surpasses the discount rate effect so that asset prices rise at the election day. Therefore, a fall in prices does not indicate that the winning party is bad for firms, or vice versa.

Despite the fact that asset prices could move up or down depending on the election result, the expected jump in stock prices is generally positive. This is true because the uncertainty about the jump makes investors demand a risk premium for holding these risky assets at the Election Day. This result is generated by the *electoral uncertainty*. Actually, if the uncertainty about the partisanship effect is the same for both parties, it is shown that the risk premium required by investors is always positive. When the election result is harder to predict, the expected jump goes up and if the market takes for granted the victory of one of the parties and the expectation is confirmed, the result has no effect on stock prices.

Prior to the election, investors require a higher risk premium than what would be the case without the *electoral uncertainty*. In general, if the signal coming from the 'political game' is very noisy, investors cannot accurately estimate the winning probabilities for both parties and demand better expected returns to hold these risky assets. Also, the required risk premium goes up as the winning probability of the party holding the best mean-variance trade-off goes down. Thus, if it is more likely that elections will move in the mean-variance direction not best for the market, the required risk premium increases.

Finally, while the focus of the paper is theoretical, some of the literature results are analyzed and discussed under the developed framework.

The literature has widely related political events to stock market movements. The effects of partisanship are discussed in some relevant articles. Santa Clara and Valkanov (2003) document that the excess return in the stock market is higher under Democratic presidencies than under Republican presidencies. Working with prediction markets tracking the election outcome, Snowberg, Wolfers and Zitzewitz (2007) find that Bush's re-election in 2004 led to modest increases in equity prices, nominal and real interest rates, oil prices and the dollar. Also, Belo, Gala and Li (2013) find that during Democratic presidencies firms with high government exposure experience higher cash flows and stock returns while the opposite holds true during Republican presidencies, pointing out a relevant channel through which the partisanship effect could work.

Other papers go in the direction to discuss the effect caused by election induced uncertainty into the stock market. Li and Born (2006) examine the influence of U.S. presidential elections on common stock returns before the election itself. Using data from public opinion polls, they find that stock market volatility increases before elections when neither of the candidates has a dominant lead in the presidential preference polls. They also find abnormally high stock market returns in the weeks preceding major elections. Pantzalis, Stungeland and Turtle (2000) find that asset valuations generally rise during the two weeks prior to a general election. They argue that political uncertainty decreases during this period, and this resolution of uncertainty leads to an increase in stock prices. They also find that the strength of these returns depend on the country's degree of political, economic and press freedom.

Pastor and Veronesi (2012) develop a general equilibrium model where the government chooses the current policy which affects firm profitability, Pastor and Veronesi (2013) develop a similar model in which stock prices respond to political news. In both works, the government tends to change the current policy when the economy is weak, but investors cannot fully anticipate which policy the government will choose. This source of uncertainty induces a risk premium that makes stocks more volatile and more correlated. As anticipated, these two works are the most closely related to the model developed, but here the main focus is on the behavior of asset prices in the presence of elections.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effects on asset prices generated by the electoral race. Section 4 develops some simulated examples where the main model predictions are presented. Section 5 discusses some literature results under the structure earlier developed. And finally, Section 6 concludes. The Appendix contains the details and proofs of the results presented.

2.2. The Model

Close to what is developed in Pastor and Veronesi (2012, 2013), the economy has a finite horizon $[0,T]$ and a continuum of firms $i \in [0,1]$. Firms i's capital at time t is denoted by B_i^t, and at time 0, $B_i^0 = 1$. The rate of return for each firm is stochastic and denoted by $d\Pi_t^i$. All profits are reinvested, then $dB_t^i = B_t^i d\Pi_t^i$. For all $t \in [0,T]$, we have:

$$d\Pi_t^i = (\mu + g_t)dt + \sigma dZ_t + \sigma_1 dZ_t^i \tag{2.1}$$

where (μ, σ, σ_1) are observable constants, Z_t is a common Brownian motion, and Z_t^i is an independent Brownian motion that is specific to firm i. The variable g_t denotes the impact of the prevailing party on power over the profitability process of each firm.

There are two parties running in the election: the incumbent party I and the challenger party C. Party I is in power from time 0 to τ. At an exogenous time $\tau \in (0,T)$, election occurs and voters choose g_t. Thus, the evolution of g_t through time is given by:

$$g_t = \begin{cases} g_I, \ \ for\ t \leq \tau \\ g_I, for\ t > \tau\ if\ party\ I\ wins \\ g_C, for\ t > \tau\ if\ party\ C\ wins \end{cases} \tag{2.2}$$

where g_I and g_C represent the impact of party I and C on firms' profitability, respectively.

The partisanship effect g_t is unknown. Investors have a probability distribution about the impact of each party over firms' profitability, where:

$$g_I \sim N(\mu_I, \sigma_I^2) \text{ and } g_C \sim N(\mu_C, \sigma_C^2). \tag{2.3}$$

This feature captures the possible uncertainty about the government impact into the stock market. This uncertainty can be generated, for example, by the market not knowing if the current government will be able to exert its promises made in the election. One channel that could prevent the government from doing this is its incapacity to pass some issues in the Congress. Then, the market assumes a distribution about the future impact. A plausible assumption is that $\sigma_C^2 > \sigma_I^2$, i.e., the challenger party carries more uncertainty about its capacity to implement its programs than the incumbent party, which people have been learning about through all the previous electoral cycle.

In the investors' side, they own the firms and are represented by a continuum of individuals who maximize expected utility derived from terminal wealth. For all $j \in [0,1]$, investor j's utility is given by:

$$u(W_T^j) = \frac{(W_T^j)^{1-\gamma}}{1-\gamma} \tag{2.4}$$

where W_T^j is investor j's wealth at time T and $\gamma > 1$ is the coefficient of relative risk aversion, These investors are equally endowed at time 0 with shares of firm stock, which pays liquidating dividends at time T. The representative conseumer then maximize its utility at any tim subject to $W_T = B_T$, where $B_T = \int_0^1 B_T^i di$ is the total wealth of the economy at time T.

2.2.1. Learning about the Election

The learning structure here is considerably different from what is presented in Pastor and Veronesi (2012, 2013). Their focus is on the fact that investors learn through time about the distribution of g_t, allowing them to have a better notion about the government effect on firms' profitability. Also, Pastor and Veronesi (2013) adds a new learning structure about the 'political cost' of adopting a new policy, which directly affects government's preferences. Here, I assume that the distribution of g_t is fixed and agents learn through time about the probability of each party winning the election. Furthermore, there is no doubt about what are the costs and benefits of changing the prevailing party in the election, i.e., the cash flow and discount rate effects are given.

Thus, suppose that, at $t = 0$, nature endows one party with the correct position for the next election , i.e., one party holds the position that is supported by the majority of the voters (or the majority of the electoral colleges, in the U.S. case). But, the parties nor the voters know the true state and information is revealed gradually and flows continuously until the election day.

During the electoral race, agents observe the process X that goes from time 0 to τ (Election Day) which represents a series of signals from the political game[10], i.e., political news, debates, scandals, etc, where X follows:

$$X_t = \mu_x dt + \sigma_x dZ_t^x \tag{2.5}$$

where Z_t^x is a Wiener process. Then, X_t is a Brownian motion with 'drift' equal to μ_x and variance equal to σ_x^2. Without loss of generality, let p_t denote the probability that investors assign at time t to party C having the correct position. To simplify, the realization $\mu_x = 1/2$ means that party C holds the correct position, while the realization $\mu_x = -1/2$ means that the incumbent party holds the correct position. The prior probability that party I or C is going to win the election is equal to $1/2$:

The learning process about the probability of each party winning the upcoming election is given by:

- **PROPOSITION 1:** Let p be the logistic function, that is,

$$p(x) = \frac{1}{1 + e^{-x}} \tag{2.6}$$

for all $x \in \mathbb{R}$, and $p(-\infty) = 0$ and $p(\infty) = 1$. Applying Bayes' rule, it follows that:

$$p_t \equiv \Pr\{\mu_x = 1/2 | X_t\} = p\left(\frac{X_t}{\sigma_x^2}\right). \tag{2.7}$$

[10] It could be used here the war of information game proposed by Gul and Pesendorfer (2012) in which parties could pay a cost to provide information in order to convince voters that they have the correct position. In this paper, I simplify the flow of information assuming an exogenous process that comes from the 'political game' which is absorbed by investors and, consequently, by the stock market.

For any time $t \leq \tau$, p_t is the estimated probability that party C will win the election. Inversely, $1 - p_t$ is the probability associated to party I winning the election.

From the flow of information, investors can estimate at any time $t \leq \tau$ the probability of each party winning the election. As the value of firms depends on which party is in power, the evolution of the above probability has an important effect over its valuation. And since investors' wealth is related to the firms' value at time T, changes in the electoral race have an impact on the expected utility causing adjustments in agents' maximization decisions.

2.3. Stock Prices

Given the implications for profitability generated by the electoral race, asset prices will respond to changes in the probability of each party winning the upcoming election. For any time $t \in [0, T]$, the total wealth in the economy is given by $B_t = \int_0^1 B_t^i di$. At time T, it can be shown that $B_T = B_\tau e^{\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}$, , where $g = g_I$ if the incumbent party wins the election or $g = g_C$ if the challenger party wins. For each firm i, its stock value is a claim on the firm's liquidating value at time T. The value of this claim at any time $t < T$ is given by:

$$S_t^i = E_t\left[\frac{\pi_T}{\pi_t} B_T^i\right] \quad (2.8)$$

where π_t denotes the state price density or the stochastic discount factor (SDF). Using the risk-free asset as numeraire, which makes a unit payoff at time T, stock prices adjust and guarantee that investors hold all firms stock. The usual finance results under market completeness yields that the stochastic discount factor is uniquely given by

$$\pi_t = \frac{1}{\lambda} E_t\left[B_T^{-\gamma}\right] \quad (2.9)$$

where λ is the Lagrange multiplier from the utility maximization problem of the representative investor.

2.3.1.Stock Prices on the Election Day

With the pricing formula established, we can now analyze what happens to stock prices on the Election Day. We know that $\pi_t = \frac{1}{\lambda} E_t\left[B_T^{-\gamma}\right]$ and $B_T = B_\tau e^{\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}$. Therefore, the SDF depends on which party wins the election. The election result makes stock prices jump and the direction depends on who wins the election.

- **PROPOSITION 2:** Suppose party C wins the election, each firm's stock return at the election day is given by

$$R_\tau^C = \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} \tag{2.10}$$

where p_τ denotes the estimated probability perceived by investors at the election day that party C will win, $F = e^{-\gamma(\mu_I-\mu_C)(T-\tau)+\frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)^2}$, $G = e^{(\mu_I-\mu_C)(T-\tau)+}\left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)^2$.

- **COROLLARY 1:** Suppose party C wins the election, the announcement return is negative, i.e., $R_\tau^C < 0$ if:

$$\mu_I - \sigma_I^2\left(\gamma - \frac{1}{2}\right)(T-\tau) > \mu_C - \sigma_C^2\left(\gamma - \frac{1}{2}\right)(T-\tau) \tag{2.11}$$

- **COROLLARY 2:** If $R_\tau^C < 0$, as $p_\tau \to 0$, the negative return at the Election Day reaches its maximum. And as $p_\tau \to 1$, $R_\tau^C \to 0$.

Combining the results in Proposition 2 and Corollary 1, at the announcement of a new party, there's a positive effect over the cash flow of the firms (μ_C) and a negative effect because of the discount rate generated by the uncertainty (σ_C^2) about this outcome. If the election changes the mean-variance trade-off from party I to party C, as in the inequality presented in equation (2.11), there's a drop in stock prices. But, a fall in stock prices at the Election Day does not necessarily mean that the winner party is the worst for firms' profitability. It could be the case that the party with higher cash flow effect, which is best for firms, brings with it a high amount of uncertainty that could be sufficient to surpass the positive effect, inducing a fall in stock prices.

Corollary 2 tells us that the possible negative effect reach its maximum if the partisanship change in the Election Day is not priced at all. Inversely, if the change is already fully priced, i.e., $p_\tau \to 1$, party C victory in the Election Day would just confirm the market prediction, what would cause no effects into the stock market.

2.3.2. Stochastic Discount Factor (SDF) Dynamics

Now, we analyze what describes the evolution of the SDF. If the election changes some stock market conditions, its effects would have significant impact prior to the election. The evolution of stock prices is closely related to the evolution of the stochastic discount factor.

- **PROPOSITION 3:** For $t \leq \tau$, the stochastic discount factor (SDF) follows the process

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t + \frac{(G_t^C - G_t^I)}{(1-p_t)G_t^C + p_t G_t^I}\sigma_x dZ_t^x + J_\pi 1_{\{t=\tau\}} \qquad (2.12)$$

where

$$G_t^C = e^{-\gamma\mu(T-t)-\gamma\mu_C(T-\tau)-\gamma\mu_I(\tau-t)+\frac{\gamma^2}{2}\left((T-\tau)^2\sigma_C^2+(\tau-t)^2\sigma_I^2\right)+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)},$$

$$G_t^I = e^{-\gamma(\mu+\mu_I)(T-t)+\frac{\gamma^2}{2}(T-t)^2\sigma_I^2+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)},$$

dZ_t^x is the Brownian motion from the signals coming from the electoral process, $1_{\{t=\tau\}}$ is an indicator equal to one for $t = \tau$ and zero otherwise, and the jump component J_π is given by

$$J_\pi = \begin{cases} J_\pi^C = \dfrac{(1-p_\tau)(1-F)}{p_\tau + (1-p_\tau)F} & if\ party\ C\ wins \\ J_\pi^I = \dfrac{p_\tau(F-1)}{p_\tau + (1-p_\tau)F} & if\ party\ I\ wins \end{cases} \qquad (2.13)$$

Finally, for $t > \tau$, the SDF follows:

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t. \qquad (2.14)$$

- **COROLLARY 3:** The expected value of the SDF jump, as perceived just before time τ, is zero:

$$E_\tau[J_\pi] = p_\tau J_\pi^C + (1 - p_\tau) J_\pi^I = 0 \quad (2.15)$$

Proposition 3 shows that the SDF dynamics is driven by shocks in the value of firms (dZ_t) and shocks in the electoral process (dZ_t^x). The first source of shocks, dZ_t, is unrelated to the political process and therefore is not affected by the election. The effect of the second source of shocks, denoted by *electoral shocks*, will depend on the sensitivity of the SDF to this specific source of uncertainty. This will depend on which party holds the best mean-variance trade-off. It can be shown that if we have, for example, $\mu_C - \frac{\gamma}{2}\sigma_C^2(T-\tau) > \mu_I - \frac{\gamma}{2}\sigma_I^2(T-\tau)$, a positive electoral shock, which increases the probability of party *C* being elected, would decrease the SDF. As $p_t \to 0$, the sensibility of the SDF to *electoral shocks* reach its maximum, i.e., the drop in the SDF is stronger when the 'good state of the world' is not priced at all. This is true because the SDF can be interpreted as the agents' marginal utility in this economy, which guarantees that any increase in the winning probability of the party with the best mean-variance trade-off would highly increase agents' expected payoff, which drives down the SDF. Since we are working with risk averse investors, they are more sensitive when their previous expected payoff is the lowest, which is the case when $p_t \to 0$. Finally, a higher uncertainty about the electoral signs (σ_x) intensifies these effects.

Using equation (2.12), it also shows that for any time $t < \tau$, we have that $E_t\left[\frac{d\pi_t}{\pi_t}\right] = 0$, and Corollary 3 guarantees that this is true also at time τ, ensuring the SDF martingale property. It is straightforward to show that if party C wins the elections, the jump in the SDF is negative if $\mu_C - \frac{\gamma}{2}\sigma_C^2(T-\tau) > \mu_I - \frac{\gamma}{2}\sigma_I^2(T-\tau)$. Thus, the SDF jump is negative in case of a policy change if the mean-variance trade-off favors party C, which increases investors expected wealth for time T and drives down the SDF.

2.3.3.Stock Price Dynamics

With The SDF dynamics defined, we can now turn to the stock price dynamics.

- **PROPOSITION 4:** For $t \leq \tau$, the return process for stock i is given by:

$$\frac{dS_t^i}{S_t^i} = \mu_{S,t}dt + \sigma dZ_t + \left(\frac{1-H}{1-p_t+p_tH} - \frac{1-M}{1-p_t+p_tM}\right)\sigma_x dZ_t^x + \sigma_1 dZ_t^i$$
$$+ J_S 1_{\{t=\tau\}} \qquad (2.16)$$

where

$$\mu_{S,t} = \gamma\sigma^2 - \left(\frac{1-H}{1-p_t+p_tH} - \frac{1-M}{1-p_t+p_tM}\right)\left(\frac{1-M}{1-p_t+p_tM}\right)\sigma_x$$

$$H = \frac{K_t^I}{K_t^C}, M = \frac{G_t^I}{G_t^C},$$

$$K_t^I = e^{(1-\gamma)(\mu+\mu_I)(T-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(T-t)^2-(1-\gamma)\gamma\frac{\sigma^2}{2}(T-t)},$$

$$K_t^C = e^{(1-\gamma)(\mu+\mu_C)(T-\tau)+(1-\gamma)(\mu+\mu_I)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2+\frac{(1-\gamma)^2}{2}\sigma_C^2(\tau-t)^2-(1-\gamma)\gamma\frac{\sigma^2}{2}(T-t)}$$

The jump component J_S is given by:

$$J_S = \begin{cases} J_S^C = \dfrac{(1-p_\tau)F(1-G)}{p_\tau+(1-p_\tau)FG} & if\ party\ C\ wins \\ J_S^I = \dfrac{p_\tau(G-1)}{p_\tau+(1-p_\tau)FG} & if\ party\ I\ wins \end{cases} \qquad (2.18)$$

Finally, for $t > \tau$, the return process is given by:

$$\frac{dS_t^i}{S_t^i} = \gamma\sigma^2 dt + \sigma dZ_t + \sigma_1 dZ_t^i \qquad (2.19)$$

Equation (2.16) shows that stock prices respond to a series of shocks as well as to the SDF. First, aggregate shocks (dZ_t) are relevant and have larger effects if the uncertainty (σ) about these shocks are higher. Second, electoral shocks (dZ_t^x) are also an important source of variation for the value of firms. And, finally, firm-specific shocks (dZ_t^i) also affects the dynamics of stock prices, but because this source of uncertainty is diversifiable across firms, they do not command a risk premium.

- **COROLLARY 4:** The conditional expected jump in stock prices, as perceived just before time τ, is given by:

$$E_\tau[J_S] = -\frac{p_\tau(1-p_\tau)(1-F)(1-G)}{p_\tau+(1-p_\tau\ )FG} \qquad (2.20)$$

- **COROLLARY 5:** For $\sigma_C^2 = \sigma_I^2$, i.e., agents have the same level of uncertainty about partisanship effects, then $E_\tau[J_S] \geq 0$ for any combinations of μ_I and μ_C. Without any restrictions, $E_\tau[J_S] < 0$ if

$$\left(\gamma - \frac{1}{2}\right)(\sigma_I^2 - \sigma_C^2)(T - \tau) < \mu_I - \mu_C < \frac{\gamma}{2}(\sigma_I^2 - \sigma_C^2)(T - \tau) \quad (2.21)$$

Since the risk-free rate is zero, the risk premium required by investors to hold risky assets in this economy is given by the expected excess return $\mu_{S,t}$ shown in Proposition 4. There are two non-fully diversifiable sources of risk in this economy. The first one, given by aggregate shocks in the firms' profitability (dZ_t), generates a positive expected return ($\gamma\sigma^2$), since they are directly correlated to investors' expected future wealth.

The second term defines the risk premium required by investors to hold assets that are subject to the political process. In general, this required risk premium is positive. But, for some combinations of parameters, this value can be negative. This fact is true because the risk premium is the return above the risk free rate required by investors to hold assets that are positively correlated to their future wealth (or negatively correlated to investors' marginal utility). If this is not the case, investors accept a negative risk premium to hold assets that protects them against an aggregate source of uncertainty.

In this case, *partisanship uncertainty* plays an important role in defining the sign of the risk premium. If the party best for firms brings with it a high amount of uncertainty sufficiently to ensure that the correlation between stock prices and the SDF have the same sign, the required risk premium will be negative. Again, it is worth noting that stock price movements are not a perfect indicator of agents' welfare. It is possible that the market face a fall in stock prices while expected wealth is rising, since the weight given by individuals and by the market to partisanship uncertainty is different. If this source of uncertainty is equalized between the two parties, the risk premium generated by electoral shocks is always positive and, in this case, stock price movements are a perfect qualitative indicator about agents' welfare.

Finally, Corollaries 4 and 5 present the dynamics of stock prices in the Election Day. Again, the same distinction between actual market returns and welfare improvements hold. But, if the partisanship uncertainty is equalized

($\sigma_C^2 = \sigma_I^2$), the conditional expected jump in stock prices, as perceived just before time τ, is greater than zero, once the uncertainty about the jump makes investors demand a risk premium for holding these risky assets in the Election Day.

2.4.Simulation Example

Now I illustrate the previous model with some simulations to shed light on the main results. Table 2.1 presents the parameters that will remain fixed through the following analysis. All variables are reported on an annual basis, except for the risk aversion parameter γ. Here, I consider that the learning process about the election begins at time $\tau - 1$ and the election result holds for 4 years ($T - \tau$). It is noteworthy that the following analysis is mainly qualitative, since the magnitude of the effects can be totally changed by the scale of the parameters.

2.4.1. Electoral Uncertainty

First, let's analyze stock prices behavior assuming that partisanship uncertainty is eliminated, which allows us to establish the results focusing only on the effects generated by the electoral process. Therefore, I assume in Figures 2.1, 2.2 and 2.3 that $\sigma_I = \sigma_C = 0$. As the cash flow effect is linear, what matters is the difference between μ_I and μ_C. Assuming that the incumbent party is best for firms, $\mu_I - \mu_C = 2\%$, Panels 1A and 1B in Figure 2.1 plots the risk premium and its components (Equation (2.17)) as a function of p_τ. The first component is constant and due to aggregate shocks, which are not affected by the electoral race. They are generated by the fact that risk averse investors demand a risk premium to hold assets that are positively correlated with their expected wealth. The second component shows that as $p_t \to 1$, i.e., as the likelihood of party C winning the election goes up, the risk premium required by investors to hold these risky assets also goes up, since the winning probability of the party with best cash flow effect is decreasing. Panels 1A and 1B show similar figures, changing only the accuracy of the electoral signs, which is smaller in the first, i.e., Panel 1A has a higher σ_x. This is intuitive because when σ_x is larger, the electoral result becomes more uncertain, once investors cannot infer a good estimation of p_t from market signs. Panels 1C and 1D are very similar to 1A and 1B, except for the fact that now the challenger party holds the best cash flow effect, where $\mu_C - \mu_I = 2\%$. The interpretation is qualitatively the same, but in this case the risk premium goes up as $p_t \to 0$.

Right before the election, investors require a risk premium for holding assets that can be affected by the election result. The size of the expected jump is given by equation (2.20). Figure 2.2 shows the expected jump as a function of p_τ. Panels 2A and 2B present a concave pattern, evidencing that regardless of which party is best for firms profitability, investors demand a risk premium to hold these risky assets and the premium is higher when the result is more uncertain. As the estimated probability converges to one of the extremes, the expected jump goes to zero, since in this case the effects of the election into the stock market are already priced. I consider another case for the risk aversion parameter where $\gamma = 3$. The figure shows that the required risk premium falls for any p_τ as investors' risk aversion decreases.

According to Corollary 1, stock prices movements when the election result is revealed depend on which party holds the best mean-variance trade-off. And, as pointed out by Corollary 2, the observed return is stronger if markets are not expecting the actual result. Given that, Figure 2.3 shows the announcement return as a function of p_τ. Panels 3A and 3B assume that $\mu_I - \mu_C = 2\%$, while Panels 3C and 3D assume that $\mu_C - \mu_I = 2\%$. The left panels 3A and 3C assume that party C wins the election, while the right panels 3B and 3D assume that party I wins. As can be seen, the size of the announcement return closely depends on the market's perception about the election. If the estimated probability goes in the direction of the election result, prices suffer smaller movements. For example, in Panel 3A, it is assumed that party I is best for firms' profitability, but party C is the winner of the election. In this case, the announcement return is negative, but the drop in stock prices is attenuated as the market better predicts the result. In the extreme, as $p_\tau \to 1$, price movements in the Election Day is null. On the other side, if the estimated probability goes to the opposite direction, the drop in prices gets higher, since the result was not expected. All other panels present a similar interpretation, changing the party with the best cash flow effect and the winner party.

2.4.2. Partisanship Uncertainty

Now I examine the asset pricing implications when the economy is also subject to *partisanship uncertainty*. In this case, voters are choosing mean-variance trade-offs, instead of just cash flow effects, as was the case in the previous section. For example, let's suppose that party C holds the cash flow advantage, where $\mu_C - \mu_I = 2\%$. But, it brings a higher uncertainty about its ability to fulfill its promises if it wins. Figure 2.4 plots the risk premium and its components as a function of p_t, assuming that σ_C gradually grows from Panel 4A to 4D and $\sigma_I = 0$.[11] Panel 4A shows that if there is no *partisanship uncertainty* the required risk premium goes up as the winning probability for Party C falls, since $\mu_C > \mu_I$. In Panels 4B and 4C both parties have a similar trade-off between the cash flow and the discount rate effects, which keeps the required risk premium almost constant. When σ_C gets higher, as in Panel 4D, the discount rate effect is strong enough to surpass the positive cash flow effect generated by Party C, generating an increase in the risk premium as $p_t \to 1$. Therefore, while on average party C is best for firms' profitability, its partisanship uncertainty is so strong that risk averse investors start avoiding this type of asset. An analogous analysis holds if party I has the cash flow advantage, but in this case the graphs would present an inverse shape.

Before the election, the expected jump also depends on how the distribution of the cash flow is dispersed. Assuming the same parameters from Figure 2.4, Figure 2.5 shows the expected jump as a function of p_τ. Again, σ_C gradually grows from Panel 5A to 5D and $\sigma_I = 0$. The figure shows that investors demand a risk premium to hold risky assets in the Election Day, but the size of the premium depends on how big is the mean-variance trade-off between the two parties. In Panel 5A, party C holds the advantage in terms of cash flow and the risk premium is relatively high when compared to Panel 5B and 5C, when the election of party C would bring with it a high amount of uncertainty. Thus, in Panels 5B and 5C investors require a small risk premium, since the mean-variance trade-off for both parties is very close. In Panel 5D the level of uncertainty about the cash flow

[11] Due to the nonlinearity of the discount rate effect, the level and the difference between σ_C and σ_I matters, while for the cash flow effect only the difference between μ_C and μ_I is relevant. But, in order to show the effects of *partisanship uncertainty* over the stock market, assuming that $\sigma_I = 0$ and that σ_C grows meet our purpose.

effect from party C is sufficient to bring the risk premium up again, similar to Panel 5A, showing that regardless of which party holds the best mean-variance trade off, the risk premium goes up when the results is more uncertain.

Still using the same parameters from Figures 2.4 and 2.5, Figure 2.6 plots the announcement return as a function of p_τ. Once more, σ_C grows from Panel 6A to 6D. Here, I consider only the case where party C wins the election. An analogous analysis can be done for party I. Panel 6A shows that, when there is no *partisanship uncertainty*, the announcement return is higher when the election result is not well priced, i.e., when $p_\tau \to 0$. When it is added some uncertainty, as in Panel 6B, the announcement returns are still positive, but smaller compared to Panel 6A. At some point, the *partisanship uncertainty* about the cash flow effect from Party C gets so high that stock prices fall, even if the winner party is better for firms' profitability on average. This is the case for Panel 6C and 6D. In Panel 6C, we have $\sigma_C = 4\%$ and $\sigma_C = 5\%$ in Panel 6D, numbers slightly different from the ones considered in Figures 2.4 and 2.5. Due to nonlinearity of the discount rate effect, it can be seen that the announcement return drastically falls as σ_C gets higher when the market does not predict very well the election result.

2.5. Discussion

In this section, some studies are interpreted using the previous model. Although this is not main focus of this article, this allows demonstration of how the model can be applied to previous results.

2.5.1. Snowberg, Wolfers and Zitzewitz (2007)

Snowberg, Wolfers and Zitzewitz (2007) analyze the effect of partisanship on equity prices and other variables. They use prediction market data which yields a market-based estimate of the probability that Bush would win the 2004 election[12] and December 2004 futures contract from a series of financial variables such as the S&P 500 and others during overnight trading. Thus, they regress changes in the S&P 500 on changes in Bush's chances of re-election using a ten minute interval to pair both sources of data. They find that Bush's re-election had

[12] They use the betting market from TradeSports (www.tradesports.com) which created a contract that would pay $10 if Bush were elected president, and zero otherwise.

a significant impact over stock markets, yielding equity prices 1.5 to 2 percent higher.

This framework can be easily connected to the previous model, since the estimated probability from prediction markets are closely related to the learning process presented in Section 2.2.1, while the partisanship effect can be interpreted as a party having the advantage over the profitability of firms. In fact, as this article works with overnight data, changes in Bush's chances of re-election could be seen in the previous model as the market trying to estimate the winning probability for both parties right before the election, i.e., p_τ.

Thus, Figure 2.7 shows what would be the model prediction of an asset that is favored by the incumbent party and which has the same market value at the end of the sample as the December 2004 futures contract of the S&P 500. As the authors found a Republican party advantage around 2%, this roughly means an increase in profitability of something close to 0.5% for the next 4 years. Therefore, working with the parameters from Table 2.1 and $\mu_I - \mu_C = 0.5\%$, while $\sigma_I = \sigma_C = 0$, Figure 2.7 shows what should be the market value predicted by the model applying the actual probability in the overnight prediction market taken from Justin Wolfers's website.[13] I assume an asset with the same market value as the futures contract of the S&P 500 at the end of the sample and go backwards discounting the final value by the estimated jump given by equation (2.18) as p_τ changes. As can be seen, the model predicts very well the evolution of the futures contract from the point where the first drop in prices around 3 p.m. occurs. This abrupt drop was caused by the fact that exit polls released at 3 p.m. suggested a Bush defeat and the price of a security paying $10 if he was re-elected fell from $5.50 to $3, an effect that was probably augmented by the uncertainty added to the election perspective since the signals coming from the electoral race became less informative. Once the uncertainty was resolved, the futures contract price stayed fairly constant, which is also the case for the model prediction. The framework presented here allows us to clearly interpret this relevant market movement in the Election Day, suggesting a channel through which political news was reflected in the stock market.

[13] Data available at http://users.nber.org/~jwolfers/data.php.

2.5.2. Li and Born (2006)

Li and Born (2006) examine the influence of U.S. presidential elections on common stock returns before the election itself. Using data from public opinion polls, they find that stock market volatility increases before elections when neither of the candidates has a dominant lead in the presidential preference polls. They also find abnormally high stock market returns in the weeks preceding major elections.

In their study, they use Gallup Poll results to measure voting intentions in the United States from 1964 to 2000. They construct a measure of U.S. presidential election uncertainty and another measure based on the democratic advantage. In the eyes of the model developed here, the election uncertainty measure created using the election polls could also be generated by the estimated probability p_t. The authors point out that if elections are too close to call, the market faces an unpredictable political regime. Consequently, volatility and returns rise. This can be seen as investors requiring a higher risk premium to hold assets subject to political uncertainty, as shown in Corollary 5. As for the volatility result, since it is added to the market another source of volatility, given by the *electoral uncertainty*, it is expected an increase in stock prices movements, once they significantly change due to requirement of different risk premiums as the election perspective changes.

2.5.3. Pantzalis, Stangeland and Turtle (2000)

Pantzalis, Stungeland and Turtle (2000) investigate the behavior of stock market indices across 33 countries around political election dates during the sample period 1974-1995. They develop an event study of stock indices' returns around international election dates. The authors mainly find that asset valuations generally rise during the two weeks prior to a general election. They argue that political uncertainty decreases during the two weeks prior to elections, and this resolution of uncertainty leads to an increase in stock prices. They also find that the strength of these returns depend on the country's degree of political, economic and press freedom.

From the model perspective, these results can be seen as the signal coming from the electoral race becoming more clear close to the election, which allows investors to make a better prediction about the upcoming result and, consequently, require smaller risk premiums to hold these risky assets, since the uncertainty about the outcome has fallen. Beyond that, the dependence on the degree of freedom can be roughly seen as how much the signals from the electoral race are reliable. In the model, this represents a high or low value for σ_x in Equation (2.5). The authors find that returns are higher in less free countries won by the opposition. This is the case in figure 2.1, Panels 1C and 1D. If the opposition is better for the market, but the signals coming from the electoral race are very noisy, the required risk premium goes up. Once this uncertainty is resolved, the increase on stock prices is higher.

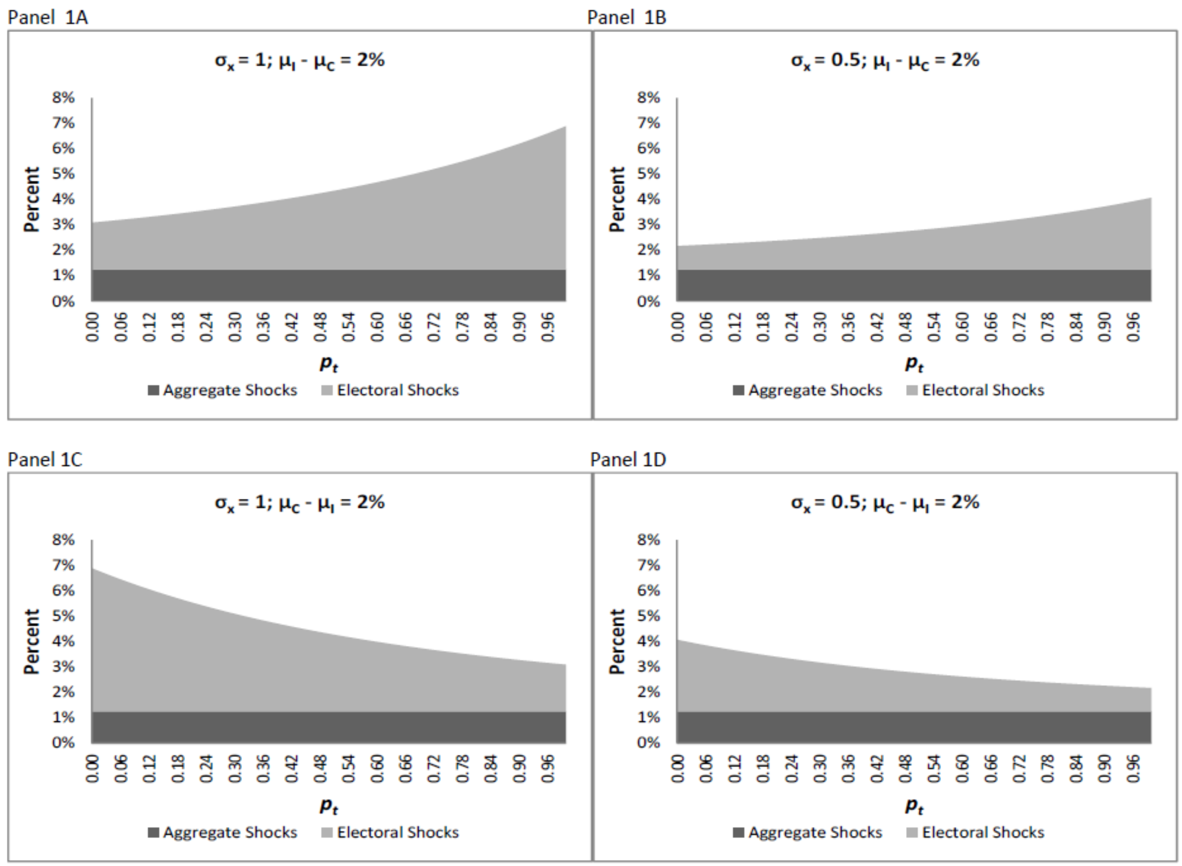
2.6. Conclusion

The present paper analyzed the effects of elections and the electoral process on asset prices. In the model, investors use signals coming from the ‘political game’ to estimate the winning probability for each party in the upcoming election. Since the party in power affects the profitability of firms, investors incorporate their expectations about political changes into prices before an election and adjust it according to changes in the election perspective. The inevitable election induced uncertainty makes investors require a risk premium to hold risky assets during the electoral race, and the size of the risk premium is larger if the winning probability of the party with the best trade-off between cash flow and discount rate effects goes down.

The model also shows that, in general, regardless of which party holds the best conditions for the stock market, the risk premium goes up when the results is more uncertain. This is true because the uncertainty about the stock prices jump in the Election Day makes investors demand a risk premium for holding these risky assets.

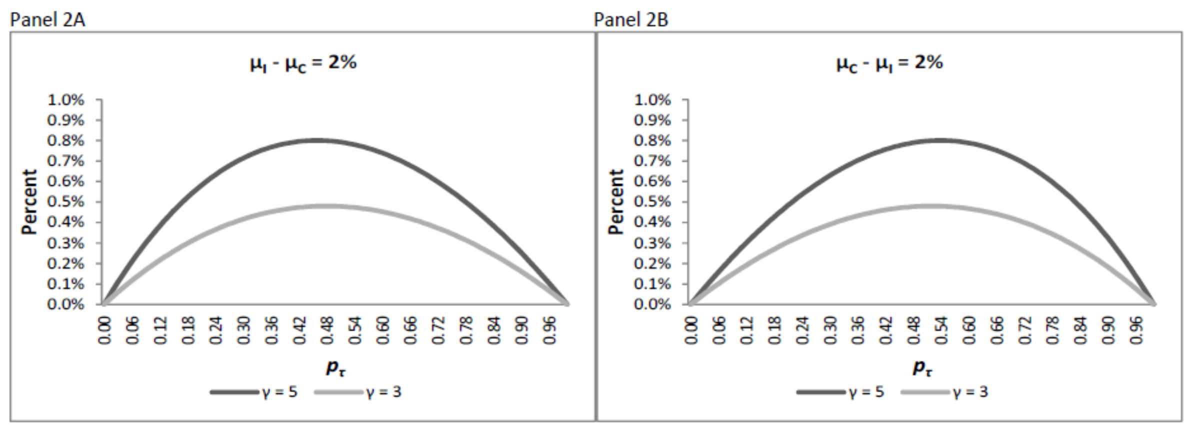
Finally, this work also presents a discussion about some results of the literature, showing how they can be interpreted under the developed framework.

Figure 2.1: The Equity Risk Premium and its components



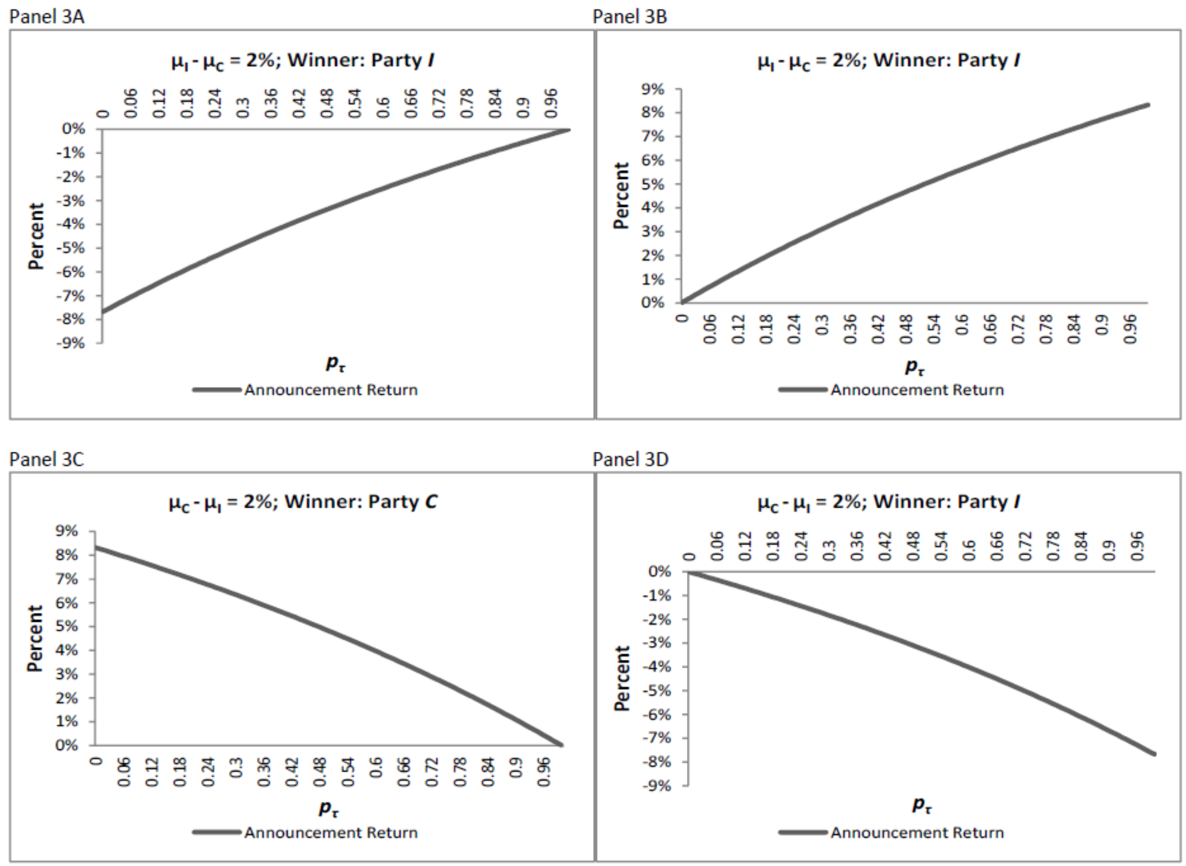
Note: This figure plots the equity risk premium as a function of the estimated probability that party C will win the upcoming election (p_t) assuming that $\mu_I - \mu_C = 2\%$ in Panels 1A and 1B, and $\mu_C - \mu_I = 2\%$ in Panels 1C and 1D. Panels 1A and 1C assume that electoral signs are less informative than Panels 1B and 1D, since σ_x is bigger in the first case.

Figure 2.2: The conditional expected jump in stock prices



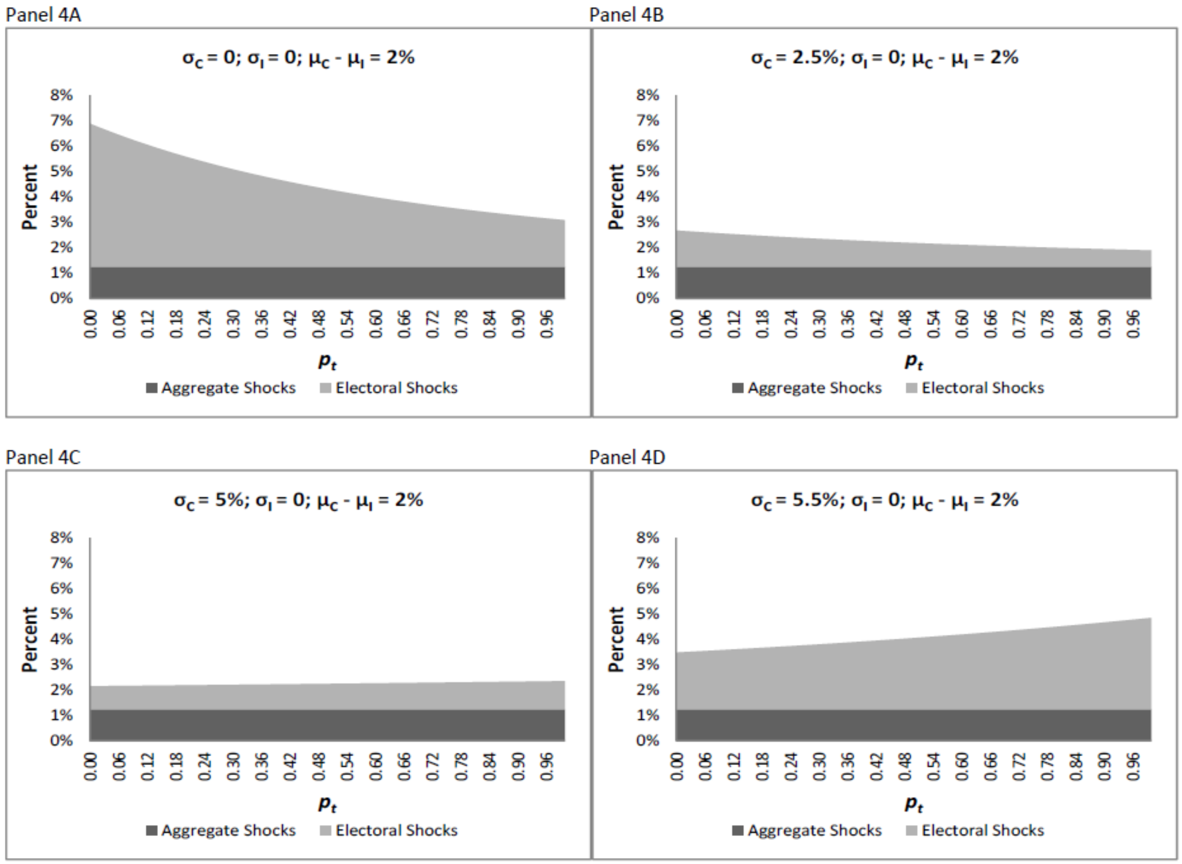
Note: This figure plots the conditional expected jump as a function of the estimated probability that party C will win right before the election (p_τ) assuming that $\mu_I - \mu_C = 2\%$ in Panel 2A and $\mu_C - \mu_I = 2\%$ in Panel 2B. In both graphs, two risk averse parameters are considered, $\gamma = 3$ and $\gamma = 5$, showing that the expected jump rises as investors' risk aversion increases.

Figure 2.3: The Announcement Returns – Electoral Uncertainty



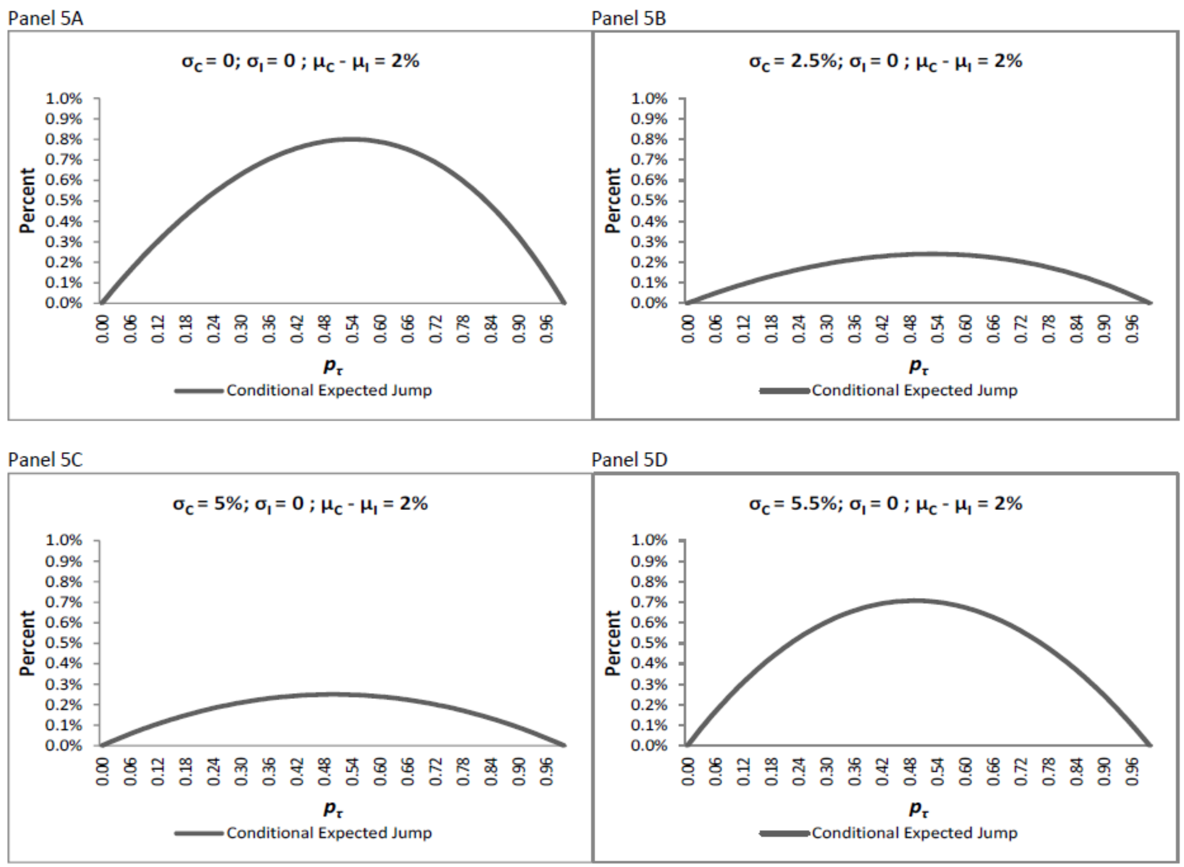
Note: This figure plots the stock prices announcement returns as a function of the estimated probability that party C will win right before the election (p_τ) assuming that $\mu_I - \mu_C = 2\%$ in Panel 3A and 3B and $\mu_C - \mu_I = 2\%$ in Panel 3C and 3D. Panels 3A and 3C assume that Party C wins the election, while Panels 3B and 3D assume that party I is the winner.

Figure 2.4: The equity risk premium and its components



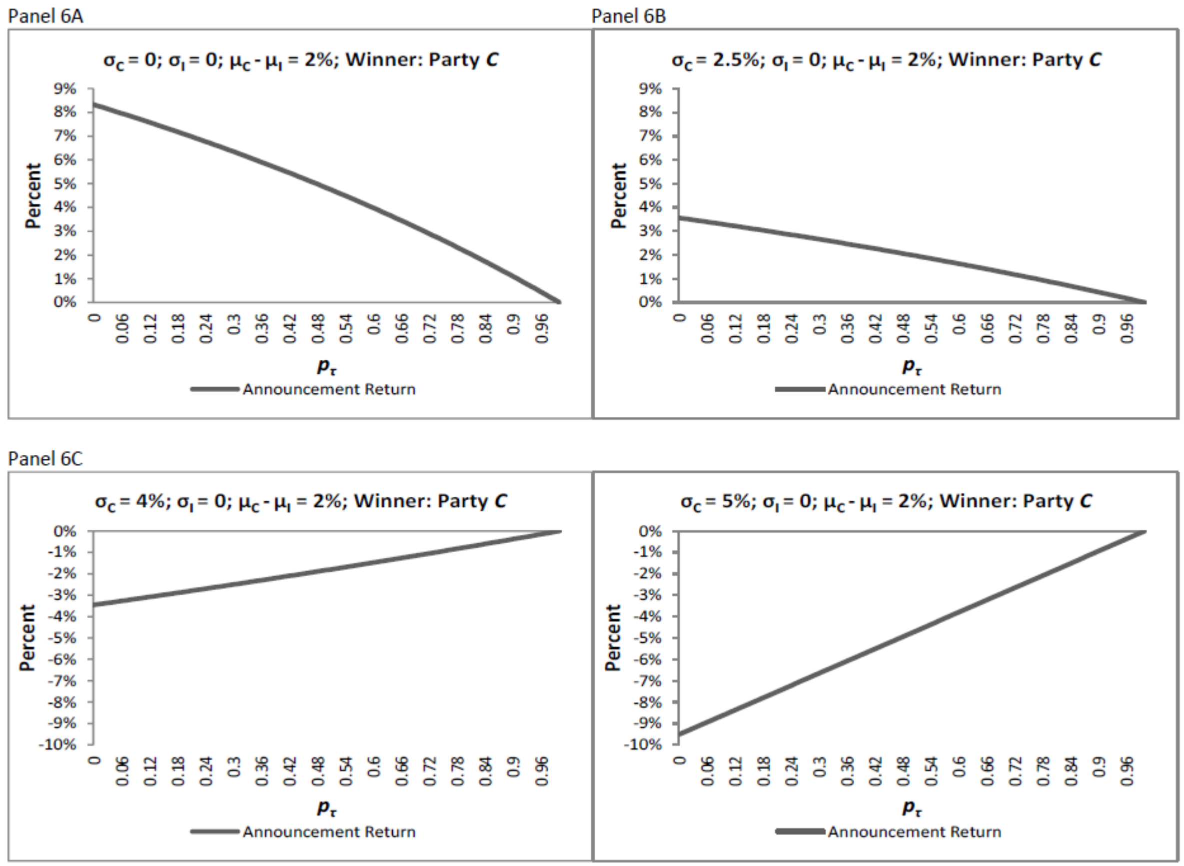
Note: This figure plots the risk premium as a function of the estimated probability that party C will win the election (p_t) assuming that $\mu_C - \mu_I = 2\%$. From Panel 4A to Panel 4D, the uncertainty about the partisanship effect from Party C gradually increases, i.e., σ_C goes from 0% to 2.5% in Panel 4B, increasing to 5% in Panel 4C and to 5.5% in Panel 4D.

Figure 2.5: The conditional expected jump in stock prices



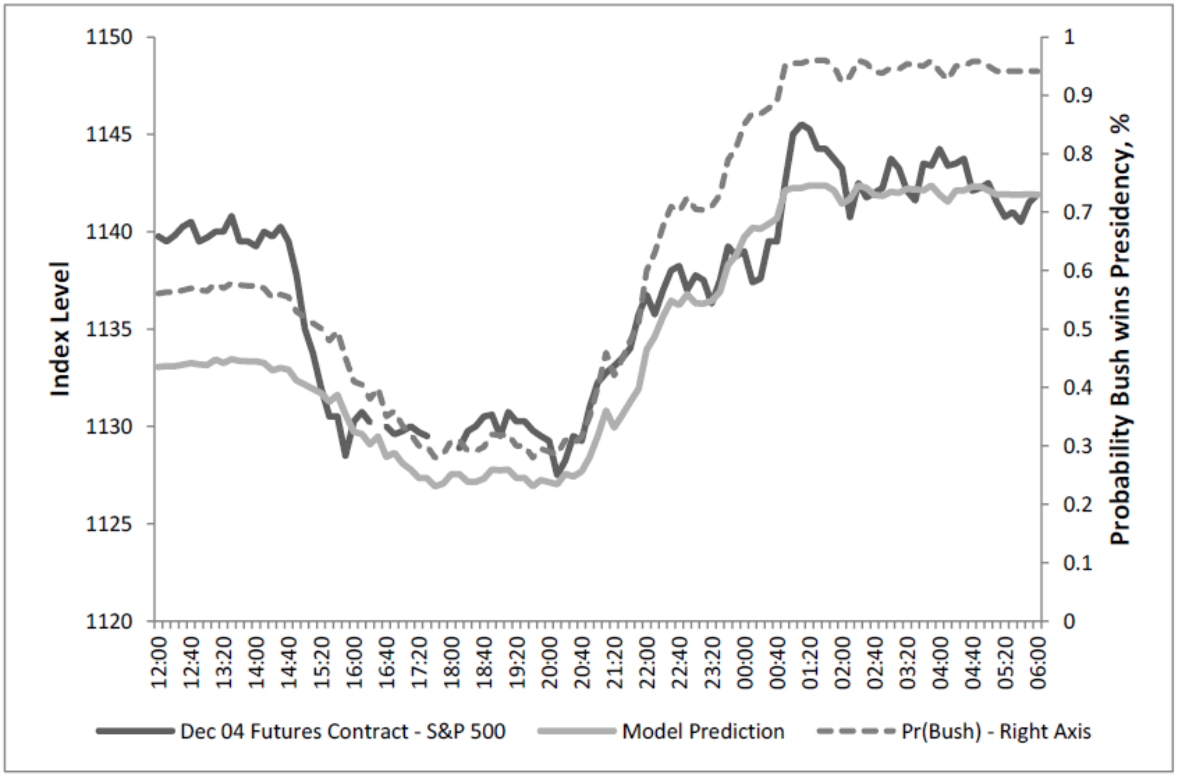
Note: This figure plots the conditional expected jump as a function of the estimated probability that party C will win right before the election (p_τ) assuming that $\mu_C - \mu_I = 2\%$. From Panel 5A to Panel 5D, the uncertainty about the partisanship effect from Party C gradually increases, i.e., σ_C goes from 0% in Panel 5A to 2.5% in Panel 5B, increasing to 5% in Panel 5C and to 5.5% in Panel 5D.

Figure 2.6: The Announcement Returns – Partisanship Uncertainty



Note: This figure plots the announcement returns as a function of the estimated probability that party C will win right before the election (p_{τ}) assuming that $\mu_C - \mu_I = 2\%$. From Panel 6A to Panel 6D, the uncertainty about the partisanship effect from Party C gradually increases, i.e., σ_C goes from 0% in Panel 6A to 2.5% in Panel 6B, increasing to 4% in Panel 6C and to 5% in Panel 6D. In all cases, party C is the election winner.

Figure 2.7: Model Prediction



Note: This figure shows the prediction market assessment of the probability of Bush's relection, the value of the S&P 500 future through noon EST on Nov 2 to 6 a.m. Nov 3 2004 and the model prediction assuming that it has the same value as the futures contract in the end of the sample. The estimated announcement return is calculated assuming that $\mu_I - \mu_C = 0.5\%$.

Table 2.1: Parameter Choices

γ	μ	σ	T	τ
5	10%	5%	8	4

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Appendix

Proofs from Section 2.2:

Section 2.2.1:

- **PROPOSITION 1:** Let p be the logistic function, that is,

$$p(x) = \frac{1}{1 + e^{-x}} \qquad (A.1)$$

for all $x \in \mathbb{R}$, and $p(-\infty) = 0$ and $p(\infty) = 1$. Applying Bayes' rule, it follows that:

$$p_t \equiv \Pr\{\mu_x = 1/2 | X_t\} = p\left(\frac{X_t}{\sigma_x^2}\right). \qquad (A.2)$$

For any time $t \leq \tau$, p_t is the estimated probability that party C will win the election. Inversely, $1 - p_t$ is the probability associated to party I winning the election.

Proof of Proposition 1: Let X_t be the state of the process X at time t. Then, applying Bayes' rule to $Pr\{\mu_x = 1/2 | X_t\}$, if follows:

$$\Pr\left\{\mu_x = \frac{1}{2} | X_t\right\} = \frac{\Pr\left\{\mu_x = \frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = \frac{1}{2}\right\}}{\Pr\left\{\mu_x = \frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = \frac{1}{2}\right\} + \Pr\left\{\mu_x = -\frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = -\frac{1}{2}\right\}}$$

$$\Pr\left\{\mu_x = \frac{1}{2} | X_t\right\} = \frac{1}{1 + \frac{\Pr\left\{\mu_x = -\frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = -\frac{1}{2}\right\}}{\Pr\left\{\mu_x = \frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = \frac{1}{2}\right\}}}$$

$$\Pr\left\{\mu_x = \frac{1}{2} | X_t\right\} = \frac{1}{1 + exp\left(ln\left(\frac{\Pr\left\{\mu_x = -\frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = -\frac{1}{2}\right\}}{\Pr\left\{\mu_x = \frac{1}{2}\right\}.\Pr\left\{X_t | \mu_x = \frac{1}{2}\right\}}\right)\right)}$$

$$\Pr\left\{\mu_x = \frac{1}{2}|X_t\right\} = \frac{1}{1 + exp\left(ln\left(\frac{\Pr\left\{\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{\mu_x = \frac{1}{2}\right\}}\right) + ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right)\right)}$$

Note that $\Pr\left\{\mu_x = \frac{1}{2}\right\} = \Pr\left\{\mu_x = -\frac{1}{2}\right\}$, then $ln\left(\frac{\Pr\left\{\mu_x=-\frac{1}{2}\right\}}{\Pr\left\{\mu_x=\frac{1}{2}\right\}}\right) = 0$. Following:

$$\Pr\left\{\mu_x = \frac{1}{2}|X_t\right\} = \frac{1}{1 + exp\left(ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right)\right)}$$

Working now only with the term inside the exponential, the normality assumption allows us to expand as follows:

$$ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right) = ln\left(\frac{\frac{1}{\sqrt{2\pi\sigma_x^2}}\,exp\left(-\left(X_t + \frac{1}{2}\right)^2/(2\sigma_x^2)\right)}{\frac{1}{\sqrt{2\pi\sigma_x^2}}\,exp\left(-\left(X_t - \frac{1}{2}\right)^2/(2\sigma_x^2)\right)}\right)$$

$$ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right) = ln\left(exp\left(\frac{\left(X_t - \frac{1}{2}\right)^2 - \left(X_t + \frac{1}{2}\right)^2}{2\sigma_x^2}\right)\right)$$

$$ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right) = ln\left(exp\left(-\frac{X_t}{\sigma_x^2}\right)\right)$$

$$ln\left(\frac{\Pr\left\{X_t|\mu_x = -\frac{1}{2}\right\}}{\Pr\left\{X_t|\mu_x = \frac{1}{2}\right\}}\right) = -\frac{X_t}{\sigma_x^2}$$

Thus, we have:

$$p_t \equiv \Pr\left\{\mu_x = \frac{1}{2}|X_t\right\} = \frac{1}{1 + exp\left(-\frac{X_t}{\sigma_x^2}\right)} = p\left(\frac{X_t}{\sigma_x^2}\right)$$

■

Proofs from Section 2.3:

- **Statement:** At time T, $B_T = B_\tau e^{\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}$.

Proof: Given that $dB_t^i = B_t^i d\Pi_t^i$ and $d\Pi_t^i = (\mu + g_t)dt + \sigma dZ_t + \sigma_1 dZ_t^1$, solving this differential equation yields the following:

$$B_T^i = B_\tau^i e^{\left(\mu+g-\frac{\sigma^2}{2}-\frac{\sigma_1^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)+\sigma_1(Z_T^i-Z_\tau^i)}$$

where $g = g_I$ if the incumbent party wins the election or $g = g_C$ if the challenger party wins. Aggregating across firms:

$$B_T = \int B_T^i di = e^{\left(\mu+g-\frac{\sigma^2}{2}-\frac{\sigma_1^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)} \int_0^1 B_\tau^i e^{\sigma_1(Z_T^i-Z_\tau^i)} di$$

The Law of Large Numbers and the independence between B_τ^i and $\left(Z_T^i - Z_\tau^i\right)$ guarantees:

$$\int_0^1 B_\tau^i e^{\sigma_1(Z_T^i-Z_\tau^i)} di = E^i\left[e^{\sigma_1(Z_T^i-Z_\tau^i)}\right] = E^i\left[B_\tau^i\right] E^i\left[e^{\sigma_1(Z_T^i-Z_\tau^i)}\right]$$

The two expectations on the right-hand side of the equation above can be written as:

$$E^i\left[B_\tau^i\right] = \int_0^1 B_\tau^i di = B_\tau \quad \text{and} \quad E^i\left[e^{\sigma_1(Z_T^i-Z_\tau^i)}\right] = e^{\frac{1}{2}\sigma_1^2(T-\tau)}$$

It follows:

$$B_T = B_\tau e^{\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}.$$

■

Section 2.3.1:

- **PROPOSITION 2:** Suppose party C wins the election, each firm's stock return at the election day is given by

$$R_\tau^C = \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} \qquad (A.3)$$

where p_τ denotes the estimated probability perceived by investors at the election day that party C will win, $F = e^{-\gamma(\mu_I-\mu_C)(T-\tau)+\frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)^2}$, $G = e^{(\mu_I-\mu_C)(T-\tau)+}\left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)^2$.

Proof of Proposition 2: Given that $\pi_t = \frac{1}{\lambda}E_t\left[B_T^{-\gamma}\right]$ and $B_T = B_\tau e^{\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}$, we then have right after the election:

$$\pi_{\tau+} = \lambda^{-1}E_{\tau+}\left[B_T^{-\gamma}\right] = \lambda^{-1}E_{\tau+}\left[B_\tau^{-\gamma}e^{-\gamma\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)-\gamma\sigma(Z_T-Z_\tau)}\right]$$

$$= \begin{cases} \pi_{\tau+}^I = \lambda^{-1}B_{\tau+}^{-\gamma}e^{\left(-\gamma(\mu+\mu_I)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_I^2}{2}(T-\tau)^2} & if\ party\ I\ wins \\ \pi_{\tau+}^C = \lambda^{-1}B_{\tau+}^{-\gamma}e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2} & if\ party\ C\ wins \end{cases}$$

Thus, the value of the stochastic discount factor (SDF) right before the election is given by

$$\pi_\tau = E_\tau[\pi_{\tau+}] = p_\tau\pi_{\tau+}^C + (1-p_\tau)\pi_{\tau+}^I$$

where p_τ denotes the estimated probability perceived by the investors at the election day that party C will win the election.

For any stock i at time t, its value right after the election is given by $S_t^i = \pi_t^{-1}E_t\left[\pi_T B_T^i\right] = \pi_t^{-1}\lambda^{-1}E_t\left[B_T^{-\gamma}B_T^i\right]$. Working with the expectation right after the election:

$$E_{\tau+}\left[B_T^{-\gamma}B_T^i\right] =$$

$$= B_{\tau+}^{-\gamma}B_{\tau+}^iE_{\tau+}\left[e^{-\gamma\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)-\gamma\sigma(Z_T-Z_\tau)}e^{\left(\mu+g-\frac{\sigma^2}{2}-\frac{\sigma_1^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)+\sigma_1\left(Z_T^i-Z_\tau^i\right)}\right]$$

$$= B_{\tau+}^{-\gamma}B_{\tau+}^iE_{\tau+}\left[e^{(1-\gamma)\left(\mu+g-\frac{\sigma^2}{2}\right)(T-\tau)+(1-\gamma)\sigma(Z_T-Z_\tau)}\right]E_{\tau+}\left[e^{-\frac{\sigma_1^2}{2}(T-\tau)+\sigma_1\left(Z_T^i-Z_\tau^i\right)}\right]$$

We have: $E_{\tau+}\left[e^{-\frac{\sigma_1^2}{2}(T-\tau)+\sigma_1\left(Z_T^i-Z_\tau^i\right)}\right] = e^{-\frac{\sigma_1^2}{2}(T-\tau)+-\frac{\sigma_1^2}{2}(T-\tau)} = e^0 = 1$. Thus, conditioning on which party wins the election, it follows:

$$E_t\left[B_T^{-\gamma}B_T^i\right]$$
$$=\begin{cases} E_{\tau+}\left[B_T^{-\gamma}B_T^i|I\right] = B_{\tau+}^{-\gamma}B_{\tau+}^i e^{\left((1-\gamma)(\mu+\mu_I)+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_I^2(T-\tau)^2} \\ E_{\tau+}\left[B_T^{-\gamma}B_T^i|C\right] = B_{\tau+}^{-\gamma}B_{\tau+}^i e^{\left((1-\gamma)(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2} \end{cases}$$

The stock values right after the election can now be written as function of which party wins: $S_{\tau+}^{i,I} = \lambda^{-1}\pi_{\tau+}^{-1}E_{\tau+}\left[B_T^{-\gamma}B_T^i|I\right]$ and $S_{\tau+}^{i,C} = \lambda^{-1}\pi_{\tau+}^{-1}E_{\tau+}\left[B_T^{-\gamma}B_T^i|C\right]$. Then, right before the election we have:

$$S_\tau^i = \frac{E_\tau\left[E_{\tau+}\left[\lambda^{-1}B_T^{-\gamma}B_T^i\right]\right]}{\pi_\tau}$$

$$S_\tau^i = \frac{p_\tau E_{\tau+}\left[\lambda^{-1}B_T^{-\gamma}B_T^i|C\right] + (1-p_\tau)E_{\tau+}\left[\lambda^{-1}B_T^{-\gamma}B_T^i|I\right]}{p_\tau\pi_{\tau+}^C + (1-p_\tau)\pi_{\tau+}^I}$$

$$S_\tau^i = \frac{p_\tau\pi_{\tau+}^C S_{\tau+}^{i,C} + (1-p_\tau)\pi_{\tau+}^I S_{\tau+}^{i,I}}{p_\tau\pi_{\tau+}^C + (1-p_\tau)\pi_{\tau+}^I}$$

Let $\omega = \frac{p_\tau\pi_{\tau+}^C}{p_\tau\pi_{\tau+}^C+(1-p_\tau)\pi_{\tau+}^I} = \frac{p_\tau}{p_\tau+(1-p_\tau)\frac{\pi_{\tau+}^I}{\pi_{\tau+}^C}}$ and

$F = \frac{\pi_{\tau+}^I}{\pi_{\tau+}^C} = e^{-\gamma(\mu_I-\mu_C)(T-\tau)+\frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)^2}$. Then $\omega = \frac{p_\tau}{p_\tau+(1-p_\tau)F}$ and the stock price right before the election can be written as:

$$S_\tau^i = \omega S_{\tau+}^{i,C} + (1-\omega)S_{\tau+}^{i,I}$$

Now, suppose that party C won the election, stock i's return after the election is given by:

$$R_\tau^C = \frac{S_{\tau+}^{i,C} - S_\tau}{S_\tau} = \frac{(1-\omega)\left(S_{\tau+}^{i,C} - S_{\tau+}^{i,I}\right)}{\omega S_{\tau+}^{i,C} + (1-\omega)S_{\tau+}^{i,I}} = \frac{(1-\omega)\left(1-\frac{S_{\tau+}^{i,I}}{S_{\tau+}^{i,C}}\right)}{\omega + (1-\omega)\frac{S_{\tau+}^{i,I}}{S_{\tau+}^{i,C}}}$$

Let $G = \frac{S_{\tau+}^{i,I}}{S_{\tau+}^{i,C}} = e^{(\mu_I-\mu_C)(T-\tau)+\left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)^2}$, the election day return is given by:

$$R_\tau^C = \frac{\left(1 - \frac{p_\tau}{p_\tau + (1-p_\tau)F}\right)(1-G)}{\frac{p_\tau}{p_\tau + (1-p_\tau)F} + \left(1 - \frac{p_\tau}{p_\tau + (1-p_\tau)F}\right)G}$$

$$R_\tau^C = \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG}$$

■

- **COROLLARY 1:** Suppose party C wins the election, the announcement return is negative, i.e., $R_\tau^C < 0$ if:

$$\mu_I - \sigma_I^2\left(\gamma - \frac{1}{2}\right)(T-\tau) > \mu_C - \sigma_C^2\left(\gamma - \frac{1}{2}\right)(T-\tau) \qquad (A.4)$$

Proof of Corollary 1: If $R_\tau^C < 0$, we have:

$$R_\tau^C = \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} < 0 \Rightarrow (1-G) < 0 \Rightarrow G > 1$$

$$G = \frac{S_{\tau+}^{i,I}}{S_{\tau+}^{i,C}} = e^{(\mu_I - \mu_C)(T-\tau) + \left(\frac{1-2\gamma}{2}\right)(\sigma_I^2 - \sigma_C^2)(T-\tau)^2} > 1$$

The inequality holds if the exponent is bigger than zero. Thus:

$$G > 1 \Leftrightarrow (\mu_I - \mu_C)(T-\tau) + \left(\frac{1-2\gamma}{2}\right)(\sigma_I^2 - \sigma_C^2)(T-\tau)^2 > 0$$

$$\Leftrightarrow \mu_I + \left(\frac{1-2\gamma}{2}\right)\sigma_I^2(T-\tau) > \mu_C + \left(\frac{1-2\gamma}{2}\right)\sigma_C^2(T-\tau)$$

$$\Leftrightarrow \mu_I - \sigma_I^2\left(\gamma - \frac{1}{2}\right)(T-\tau) > \mu_C - \sigma_C^2\left(\gamma - \frac{1}{2}\right)(T-\tau)$$

■

- **COROLLARY 2:** If $R_\tau^C < 0$, as $p_\tau \to 0$, the negative return at the Election Day reaches its maximum. And as $p_\tau \to 1$, $R_\tau^C \to 0$.

Proof of Corollary 2: As $p_\tau \to 1$, it is trivial to show that $R_\tau^C \to 0$. For the case where $p_\tau \to 0$, we would have:

$$\lim_{p_\tau \to 0} R_\tau^C = \frac{1-G}{G}$$

Under what conditions we have $\lim_{p_\tau \to 0} R_\tau^C > R_\tau^C$:

$$\frac{1-G}{G} > \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} \Rightarrow p_\tau > 1$$

which is never true. Thus, we have the $\frac{1-G}{G}$ is always smaller thanR_τ^C for any $p_\tau \in (0,1]$.

■

Section 2.3.2:

- **PROPOSITION 3:** For $t \leq \tau$, the stochastic discount factor (SDF) follows the process

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t + \frac{(G_t^C - G_t^I)}{(1-p_t)G_t^C + p_t G_t^I}\sigma_x dZ_t^x + J_\pi 1_{\{t=\tau\}} \qquad (A.5)$$

where

$G_t^C = e^{-\gamma\mu(T-t)-\gamma\mu_C(T-\tau)-\gamma\mu_I(\tau-t)+\frac{\gamma^2}{2}\left((T-\tau)^2\sigma_C^2+(\tau-t)^2\sigma_I^2\right)+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}$,

$G_t^I = e^{-\gamma(\mu+\mu_I)(T-t)+\frac{\gamma^2}{2}(T-t)^2\sigma_I^2+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}$,

dZ_t^x is the Brownian motion from the signals coming from the electoral process, $1_{\{t=\tau\}}$ is an indicator equal to one for $t = \tau$ and zero otherwise, and the jump component J_π is given by

$$J_\pi = \begin{cases} J_\pi^C = \dfrac{(1-p_\tau)(1-F)}{p_\tau + (1-p_\tau)F} & if\ party\ C\ wins \\ J_\pi^I = \dfrac{p_\tau(F-1)}{p_\tau + (1-p_\tau)F} & if\ party\ I\ wins \end{cases} \qquad (A.6)$$

Finally, for $t > \tau$, the SDF follows:

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t. \qquad (A.7)$$

Proof of Proposition 3: Given the martingale property of the SDF, we know that:

$\pi_t = E_t[\pi_{\tau+}]$

$\pi_t = p_t E_t[\pi_{\tau+}|C] + (1-p_t)E_t[\pi_{\tau+}|I]$

$\pi_t = p_t E_t[\pi_{\tau+}^C] + (1-p_t)E_t[\pi_{\tau+}^I]$

We have that:

$$\pi_{\tau+}^C = \lambda^{-1} B_{\tau+}^{-\gamma} e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2}$$

$$\pi_{\tau+}^I = \lambda^{-1} B_{\tau+}^{-\gamma} e^{\left(-\gamma(\mu+\mu_I)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_I^2}{2}(T-\tau)^2}$$

Taking expectations:

$$E_t[\pi_{\tau+}^C] = \lambda^{-1} e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2} E_t\left[B_{\tau+}^{-\gamma}|C\right]$$

$$E_t[\pi_{\tau+}^I] = \lambda^{-1} e^{\left(-\gamma(\mu+\mu_I)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_I^2}{2}(T-\tau)^2} E_t\left[B_{\tau+}^{-\gamma}|I\right]$$

By continuity, $B_{\tau+} = B_\tau$ and the value of B_τ is independent of which party wins the election. Thus, $E_t\left[B_{\tau+}^{-\gamma}|C\right] = E_t\left[B_{\tau+}^{-\gamma}|I\right] = E_t\left[B_\tau^{-\gamma}\right]$.

As party I is in power before the election, we have $\frac{B_\tau}{B_t} = e^{\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(T-\tau)+\sigma(Z_T-Z_\tau)}$. Let $b_\tau \equiv \ln(B_\tau)$, using Ito's lemma:

$$db_t = \frac{1}{B_t}dB_t - \frac{1}{2B_t^2}(dB_t)^2$$

Given that $dB_t = B_t d\Pi_t$,

$$db_t = \left(\mu + \mu_I - \frac{\sigma^2}{2}\right)dt + \sigma dZ_t$$

Integrating from t to τ:

$$b_\tau = b_t + \left(\mu + \mu_I - \frac{\sigma^2}{2}\right)dt + \sigma(Z_\tau - Z_t)$$

Writing $E_t\left[B_\tau^{-\gamma}\right] = E_t[(e^{b_\tau})^{-\gamma}] = E_t[e^{-\gamma b_\tau}]$, we have:

$$E_t\left[B_\tau^{-\gamma}\right] = e^{-\gamma E_t[b_\tau]+\frac{\gamma^2}{2}V_t[b_\tau]}$$

$$E_t\left[B_\tau^{-\gamma}\right] = e^{-\gamma b_t-\gamma\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\frac{\gamma^2}{2}\sigma_I^2(\tau-t)+\frac{\gamma^2}{2}\sigma^2(\tau-t)}$$

Turning back to π_t,

$$\pi_t = p_t\lambda^{-1} e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2} \times$$
$$\times\, e^{-\gamma b_t-\gamma\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\frac{\gamma^2}{2}\sigma_I^2(\tau-t)+\frac{\gamma^2}{2}\sigma^2(\tau-t)} +$$

$$+(1-p_t)\lambda^{-1}e^{\left(-\gamma(\mu+\mu_I)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_I^2}{2}(T-\tau)^2}\times$$

$$\times\, e^{-\gamma b_t-\gamma\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\frac{\gamma^2}{2}\sigma_I^2(\tau-t)+\frac{\gamma^2}{2}\sigma^2(\tau-t)}.$$

$$\pi_t=\lambda^{-1}B_t^{-\gamma}p_t e^{-\gamma\mu(T-t)-\gamma\mu_C(T-\tau)-\gamma\mu_I(\tau-t)+\frac{\gamma^2}{2}\left((T-\tau)^2\sigma_C^2+(\tau-t)^2\sigma_I^2\right)+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}+$$

$$+\lambda^{-1}B_t^{-\gamma}(1-p_t)\, e^{-\gamma(\mu+\mu_I)(T-t)+\frac{\gamma^2}{2}(T-t)^2\sigma_I^2+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}$$

Let:

$$G_t^C=e^{-\gamma\mu(T-t)-\gamma\mu_C(T-\tau)-\gamma\mu_I(\tau-t)+\frac{\gamma^2}{2}\left((T-\tau)^2\sigma_C^2+(\tau-t)^2\sigma_I^2\right)+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}$$

$$G_t^I=e^{-\gamma(\mu+\mu_I)(T-t)+\frac{\gamma^2}{2}(T-t)^2\sigma_I^2+\gamma(1+\gamma)\frac{\sigma^2}{2}(T-t)}$$

We have:

$$\pi_t=\lambda^{-1}B_t^{-\gamma}(p_tG_t^C+(1-p_t)G_t^I).$$

Let $\Omega(p_t,t)=p_tG_t^C+(1-p_t)G_t^I$, it follows:

$$\pi_t=E_t[\pi_{\tau+}]=\lambda^{-1}B_t^{-\gamma}\Omega(p_t,t)$$

We now have the SDF at any point in time $<\tau$. Applying Ito's lemma to the SDF allows us to obtain the evolution of the SDF through time. The martingale property of the SDF guarantees that its drift is equal to zero. Another relevant point is that the SDF jumps at time τ depending on the election results, which add one final term to the differential equation. Omitting λ^{-1}, which would drop out later, we have:

$$d\pi_t=\frac{\partial\pi_t}{\partial B_t}dB_t+\frac{\partial\pi_t}{\partial p_t}dp_t+\frac{\partial\pi_t}{\partial t}dt+\frac{1}{2}\frac{\partial^2\pi_t}{\partial B_t^2}(dB_t)^2+\frac{1}{2}\frac{\partial^2\pi_t}{\partial p_t^2}(dp_t)^2+$$

$$+\frac{\partial^2\pi_t}{\partial B_t\partial\pi_t}dB_tdp_t+J_\pi 1_{\{t=\tau\}}$$

First, let's derive dp_t. We know that $X_t=\mu_xdt+\sigma_xdZ_t^x$ and $p_t=p(X_t)=\frac{1}{1+e^{-X_t}}$.

The partial derivatives are given by:

$$\frac{\partial p(X_t)}{\partial t}=0$$

$$\frac{\partial p(X_t)}{\partial X_t}=\frac{1}{(1+e^{-X_t})^2}e^{-X_t}=\frac{1}{1+e^{-X_t}}\left(1-\frac{1}{1+e^{-X_t}}\right)=p(X_t)\big(1-p(X_t)\big)$$

$$\frac{\partial^2 p_t}{\partial X_t^2}=\frac{\partial p(X_t)}{\partial X_t}\big(1-p(X_t)\big)-p(X_t)\frac{\partial p(X_t)}{\partial X_t}=\frac{\partial p(X_t)}{\partial X_t}(1-2p(X_t))$$

$$=p(X_t)\big(1-p(X_t)\big)(1-2p(X_t)).$$

Thus:

$$dp_t = \left[\frac{\partial p_t}{\partial t} + \mu_x \frac{\partial p_t}{\partial X_t} + \frac{1}{2}\sigma_x^2 \frac{\partial^2 p_t}{\partial X_t^2}\right] dt + \sigma_x \frac{\partial p_t}{\partial X_t} dZ_t^x$$

$$dp_t = p_t(1-p_t)\left[\left(\mu_x + \frac{1}{2}\sigma_x^2(1-2p_t)\right)dt + \sigma_x dZ_t^x\right]$$

Back to $d\pi_t$:

$$d\pi_t = -\gamma B_t^{-\gamma}\Omega(p_t,t)\frac{dB_t}{B_t} + B_t^{-\gamma}\Omega(p_t,t)\frac{1}{\Omega(p_t,t)}\frac{\partial\Omega(p_t,t)}{\partial p_t} dp_t +$$

$$+B_t^{-\gamma}\Omega(p_t,t)\frac{1}{\Omega(p_t,t)}\frac{\partial\Omega(p_t,t)}{\partial t} dt + \frac{1}{2}\gamma(1+\gamma)B_t^{-\gamma}\Omega(p_t,t)\left(\frac{dB_t}{B_t}\right)^2 +$$

$$+\frac{1}{2}B_t^{-\gamma}\Omega(p_t,t)\frac{1}{\Omega(p_t,t)}\frac{\partial^2\Omega(p_t,t)}{\partial p_t^2}(dp_t)^2 -$$

$$-\gamma B_t^{-\gamma}\Omega(p_t,t)\frac{1}{\Omega(p_t,t)}\frac{\partial\Omega(p_t,t)}{\partial p_t} dB_t dp_t + J_\pi 1_{\{t=\tau\}}$$

Because the SDF is a martingale, the terms related to dt will sum zero. Then, we have:

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t + \frac{1}{\Omega(p_t,t)}\frac{\partial\Omega(p_t,t)}{\partial p_t} p_t(1-p_t)\sigma_x dZ_t^x + J_\pi 1_{\{t=\tau\}}$$

Since $\Omega(p_t,t) = p_t G_t^C + (1-p_t)G_t^I$, it follows that $\frac{\partial\Omega(p_t,t)}{\partial p_t} = G_t^C - G_t^I$. Substituting in $\frac{d\pi_t}{\pi_t}$:

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t + \frac{(G_t^C - G_t^I)}{(1-p_t)G_t^C + p_t G_t^I}\sigma_x dZ_t^x + J_\pi 1_{\{t=\tau\}}$$

Now, taking a closer look at $J_\pi 1_{\{t=\tau\}}$, if the challenger party wins the election, we would have:

$$J_{\pi,\tau}^C = \frac{\pi_{\tau+}^C}{\pi_\tau} - 1 = \frac{\pi_{\tau+}^C}{p_\tau\pi_{\tau+}^C + (1-p_\tau)\pi_{\tau+}^I} - 1 = \frac{1}{p_\tau + (1-p_\tau)\frac{\pi_{\tau+}^I}{\pi_{\tau+}^C}}$$

Previously, we defined $F = \frac{\pi_{\tau+}^I}{\pi_{\tau+}^C}$:

$$J_{\pi,\tau}^C = \frac{1}{p_\tau + (1-p_\tau)F} - 1 = \frac{(1-p_\tau)(1-F)}{p_\tau + (1-p_\tau)F}$$

Following, if party I wins the election, then the jump is given by:

$$J_{\pi,\tau}^{I} = \frac{\pi_{\tau+}^{I}}{\pi_{\tau}} - 1 = \frac{\pi_{\tau+}^{I}}{p_{\tau}\pi_{\tau+}^{C} + (1-p_{\tau})\pi_{\tau+}^{I}} - 1 = \frac{\frac{\pi_{\tau+}^{I}}{\pi_{\tau+}^{C}}}{p_{\tau} + (1-p_{\tau})\frac{\pi_{\tau+}^{I}}{\pi_{\tau+}^{C}}} - 1$$

$$J_{\pi,\tau}^{I} = \frac{F}{p_{\tau} + (1-p_{\tau})F} - 1 = \frac{p_{\tau}(F-1)}{p_{\tau} + (1-p_{\tau})F}$$

Finally, for any time $t > \tau$, electoral shocks do not occur and the evolution of the SDF is given by:

$$\frac{d\pi_t}{\pi_t} = -\gamma\sigma dZ_t.$$

■

- **COROLLARY 3:** The expected value of the SDF jump, as perceived just before time τ, is zero:

$$E_{\tau}[J_{\pi}] = p_{\tau}J_{\pi}^{C} + (1-p_{\tau})J_{\pi}^{I} = 0 \qquad (A.8)$$

Proof of Corollary 3: For any time $t < \tau$, we have:

$$E_{\tau}[J_{\pi}] = p_{\tau}E_{t}[J_{\pi}^{C}] + (1-p_{\tau})E_{t}[J_{\pi}^{I}]$$

$$E_{\tau}[J_{\pi}] = p_{t}\frac{(1-p_{\tau})(1-F)}{p_{\tau} + (1-p_{\tau})F} + (1-p_{t})\frac{p_{\tau}(F-1)}{p_{\tau} + (1-p_{\tau})F}$$

$$E_{\tau}[J_{\pi}] = 0.$$

■

Section 2.3.3:

- **PROPOSITION 4:** For $t \leq \tau$, the return process for stock i is given by:

$$\frac{dS_t^i}{S_t^i} = \mu_{S,t}dt + \sigma dZ_t + \left(\frac{1-H}{1-p_t+p_tH} - \frac{1-M}{1-p_t+p_tM}\right)\sigma_x dZ_t^x + \sigma_1 dZ_t^i + J_S 1_{\{t=\tau\}} \qquad (A.9)$$

where

$$\mu_{S,t} = \gamma\sigma^2 - \left(\frac{1-H}{1-p_t+p_tH} - \frac{1-M}{1-p_t+p_tM}\right)\left(\frac{1-M}{1-p_t+p_tM}\right)\sigma_x$$

$$H = \frac{K_t^I}{K_t^C}, M = \frac{G_t^I}{G_t^C},$$

$$K_t^I = e^{(1-\gamma)(\mu+\mu_I)(T-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(T-t)^2-(1-\gamma)\gamma\frac{\sigma^2}{2}(T-t)},$$

$$K_t^C = e^{(1-\gamma)(\mu+\mu_C)(T-\tau)+(1-\gamma)(\mu+\mu_I)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2+\frac{(1-\gamma)^2}{2}\sigma_C^2(\tau-t)^2-(1-\gamma)\gamma\frac{\sigma^2}{2}(T-t)}$$

The jump component J_S is given by:

$$J_S = \begin{cases} J_S^C = \dfrac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} & if\ party\ C\ wins \\ J_S^I = \dfrac{p_\tau(G-1)}{p_\tau + (1-p_\tau)FG} & if\ party\ I\ wins \end{cases} \qquad (A.10)$$

Finally, for $t > \tau$, the return process is given by:

$$\frac{dS_t^i}{S_t^i} = \gamma\sigma^2 dt + \sigma dZ_t + \sigma_1 dZ_t^i \qquad (A.11)$$

Proof of Proposition 4: For any time $t \le \tau$, we have $S_t^i = \frac{E_t[\pi_{\tau+}S_{\tau+}^i]}{\pi_t}$, where $\pi_t = \lambda^{-1}B_t^{-\gamma}\Omega(p_t,t)$. Working with the numerator, we have:

$$E_t[\pi_{\tau+}S_{\tau+}^i] = E_t[p_\tau \pi_{\tau+}^C S_{\tau+}^{i,C} + (1-p_\tau)\pi_{\tau+}^I S_{\tau+}^{i,I}]$$

$$E_t[\pi_{\tau+}S_{\tau+}^i] = p_t E_t[\pi_{\tau+}^C S_{\tau+}^{i,C}] + (1-p_t)E_t[\pi_{\tau+}^I S_{\tau+}^{i,I}]$$

The price for stock i right after the election depends on the winner:

$$S_{\tau+}^i = \pi_{\tau+}^{-1}E_{\tau+}[\pi_T B_T^i] = \pi_{\tau+}^{-1}\lambda^{-1}E_{\tau+}[B_T^{-\gamma}B_T^i]$$

If party C wins, we have:

$$S_{\tau+}^{i,C} = \frac{1}{\pi_{\tau+}^C}\left[\lambda^{-1}B_{\tau+}^{-\gamma}B_{\tau+}^i e^{\left((1-\gamma)(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2}\right]$$

$$S_{\tau+}^{i,C} = \frac{\lambda^{-1}B_{\tau+}^{-\gamma}B_{\tau+}^i e^{\left((1-\gamma)(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2}}{\lambda^{-1}B_{\tau+}^{-\gamma}e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2}}$$

$$S_{\tau+}^{i,C} = B_{\tau+}^i e^{(\mu+\mu_C-\gamma\sigma^2)(T-\tau)+\frac{(1-2\gamma)}{2}\sigma_C^2(T-\tau)^2}$$

Analogously:

$$S_{\tau+}^{i,I} = B_{\tau+}^i e^{(\mu+\mu_I-\gamma\sigma^2)(T-\tau)+\frac{(1-2\gamma)}{2}\sigma_I^2(T-\tau)^2}$$

Back at $E_t[\pi_{\tau+}^C S_{\tau+}^{i,C}]$ and $E_t[\pi_{\tau+}^I S_{\tau+}^{i,I}]$:

$$E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right] = E_t\Bigg[\lambda^{-1}B_{\tau+}^{-\gamma}e^{\left(-\gamma(\mu+\mu_C)+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2\sigma_C^2}{2}(T-\tau)^2}\times$$

$$\times B^i_{\tau+}e^{\left(\mu+\mu_C-\gamma\sigma^2\right)(T-\tau)+\frac{(1-2\gamma)}{2}\sigma_C^2(T-\tau)^2}\Bigg]$$

$$E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right] = \lambda^{-1}e^{\left((1-\gamma)(\mu+\mu_C)+\frac{\gamma(\gamma-1)}{2}\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2}E_t\left[B_{\tau+}^{-\gamma}B^i_{\tau+}\right]$$

For $t<\tau$, the value of B^i_τ does not depend on which party wins the election, since party I is in power from t to τ. Thus:

$$B^i_{\tau+} = B^i_\tau = B^i_t e^{\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\sigma(Z_\tau-Z_t)-\frac{1}{2}\sigma_1^2(\tau-t)+\sigma_1\left(Z^i_\tau-Z^i_t\right)}$$

$$= B^i_t\left(\frac{B_\tau}{B_t}\right)e^{-\frac{1}{2}\sigma_1^2(\tau-t)+\sigma_1\left(Z^i_\tau-Z^i_t\right)}$$

Thus:

$$E_t\left[B_{\tau+}^{-\gamma}B^i_{\tau+}\right] = \frac{B^i_t}{B_t}E_t\left[B_\tau^{1-\gamma}e^{-\frac{1}{2}\sigma_1^2(\tau-t)+\sigma_1\left(Z^i_\tau-Z^i_t\right)}\right]$$

$$= \frac{B^i_t}{B_t}E_t\left[B_\tau^{1-\gamma}\right]$$

$$= \frac{B^i_t}{B_t}E_t\left[B_t^{1-\gamma}e^{(1-\gamma)\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+(1-\gamma)\sigma(Z_\tau-Z_t)}\right]$$

$$= B^i_t B_t^{-\gamma}e^{(1-\gamma)\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(\tau-t)^2+\frac{(1-\gamma)^2}{2}\sigma^2(\tau-t)}$$

Using $E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right]$ again, we have:

$$E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right] = \lambda^{-1}e^{\left((1-\gamma)(\mu+\mu_C)+\frac{\gamma(\gamma-1)}{2}\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2}\times$$

$$\times B^i_t B_t^{-\gamma}e^{(1-\gamma)\left(\mu+\mu_I-\frac{\sigma^2}{2}\right)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(\tau-t)^2+\frac{(1-\gamma)^2}{2}\sigma^2(\tau-t)}$$

$$E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right] = \lambda^{-1}B^i_t B_t^{-\gamma}e^{(1-\gamma)(\mu+\mu_C)(T-\tau)+(1-\gamma)(\mu+\mu_I)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2}\times$$

$$\times e^{\frac{(1-\gamma)^2}{2}\sigma_I^2(\tau-t)^2-(1-\gamma)\gamma\sigma^2(T-t)}$$

Analogously for party I:

$$E_t\left[\pi^C_{\tau+}S^{i,C}_{\tau+}\right] = \lambda^{-1}B^i_t B_t^{-\gamma}e^{(1-\gamma)(\mu+\mu_I)(T-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(T-\tau)^2-(1-\gamma)\gamma\sigma^2(T-t)}$$

Let:

$$K^C_t = e^{(1-\gamma)(\mu+\mu_C)(T-\tau)+(1-\gamma)(\mu+\mu_I)(\tau-t)+\frac{(1-\gamma)^2}{2}\sigma_C^2(T-\tau)^2+\frac{(1-\gamma)^2}{2}\sigma_I^2(\tau-t)^2-(1-\gamma)\gamma\sigma^2(T-t)}$$

$K_t^I = e^{(1-\gamma)(\mu+\mu_I)(T-t)+\frac{(1-\gamma)^2}{2}\sigma_I^2(T-\tau)^2-(1-\gamma)\gamma\sigma^2(T-t)}$

We can now write:

$E_t\left[\pi_{\tau+}S_{\tau+}^i\right] = p_t\lambda^{-1}B_t^iB_t^{-\gamma}K_t^C + (1-p_t)\lambda^{-1}B_t^iB_t^{-\gamma}K_t^I$

Back to S_t^i:

$$S_t^i = \frac{E_t\left[\pi_{\tau+}S_{\tau+}^i\right]}{\pi_t} = \frac{p_t\lambda^{-1}B_t^iB_t^{-\gamma}K_t^C + (1-p_t)\lambda^{-1}B_t^iB_t^{-\gamma}K_t^I}{\lambda^{-1}B_t^{-\gamma}\Omega(p_t,t)}$$

$$S_t^i = B_t^i\left[\frac{p_tK_t^C + (1-p_t)K_t^I}{\Omega(p_t,t)}\right]$$

Finally, let $\Phi(p_t,t) = p_tK_t^C + (1-p_t)K_t^I$, we have:

$$S_t^i = B_t^i\frac{\Phi(p_t,t)}{\Omega(p_t,t)}$$

For $t < \tau$, using the fact that shocks to firms' profitability (dZ_t and dZ_t^i) are orthogonal to electoral shocks (dZ_t^x), we can apply Ito's Lemma to $S_t^i = B_t^i\frac{\Phi(p_t,t)}{\Omega(p_t,t)}$ to get the volatility term. This step yields:

$$\frac{dS_t^i}{S_t^i} = \mu_{S,t}dt + \sigma dZ_t + \sigma_1 dZ_t^i + \left(\frac{1}{\Phi(p_t,t)}\frac{\partial\Phi(p_t,t)}{\partial p_t} - \frac{1}{\Omega(p_t,t)}\frac{\partial\Omega(p_t,t)}{\partial p_t}\right)\sigma_x dZ_t^x$$

$$\frac{dS_t^i}{S_t^i} = \mu_{S,t}dt + \sigma dZ_t + \sigma_1 dZ_t^i + \left(\frac{K_t^C - K_t^I}{(1-p_t)K_t^C + p_tK_t^I} - \frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_tG_t^I}\right)\sigma_x dZ_t^x$$

And, the drift term, which can be interpreted as the risk premium required by investors to hold the risky asset, is given by $\mu_S = -Cov\left(\frac{d\pi_t}{\pi_t},\frac{dS_t^i}{S_t^i}\right)$:

$$\mu_S = -Cov\left(\frac{d\pi_t}{\pi_t},\frac{dS_t^i}{S_t^i}\right)$$

$$\mu_S = -Cov\Bigg(-\gamma dZ_t + \frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_tG_t^I}\sigma_x dZ_t^x,$$
$$[\circ]dt + \sigma dZ_t + \sigma_1 dZ_t^i + \left(\frac{K_t^C - K_t^I}{(1-p_t)K_t^C + p_tK_t^I} - \frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_tG_t^I}\right)\sigma_x dZ_t^x\Bigg)$$

$$\mu_S = \Bigg[\gamma\sigma^2 - \Bigg(\frac{K_t^C - K_t^I}{(1-p_t)K_t^C + p_tK_t^I}$$
$$- \frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_tG_t^I}\Bigg)\left(\frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_tG_t^I}\right)\sigma_x\Bigg]dt + \sigma dZ_t + \sigma_1 dZ_t^x$$

$$+\left(\frac{K_t^C - K_t^I}{(1-p_t)K_t^C + p_t K_t^I} - \frac{G_t^C - G_t^I}{(1-p_t)G_t^C + p_t G_t^I}\right)\sigma_x dZ_t^x + J_S 1_{\{t=\tau\}}$$

Taking a closer look at $J_S 1_{\{t=\tau\}}$, if party C wins the election, we know from Proposition 2 that:

$$J_{S,\tau}^C = R_\tau^C = \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG}$$

Now, if party I stays in power, then we would have:

$$J_{S,\tau}^I = R_\tau^I = \frac{S_{\tau+}^{i,I} - S_\tau^i}{S_\tau^i}$$

Using the following facts: $S_\tau^i = \omega S_{\tau+}^{i,C} + (1-\omega)S_{\tau+}^{i,I}$, $\omega = \frac{p_\tau}{p_\tau + (1-p_\tau)F}$ and $G = \frac{S_{\tau+}^{i,I}}{S_{\tau+}^{i,C}}$, we can write:

$$J_{S,\tau}^I = R_\tau^I = \frac{p_\tau(G-1)}{p_\tau + (1-p_\tau)FG}$$

Finally, for $t > \tau$, electoral shocks do not occur and the evolution of stock prices is given by:

$$\frac{dS_t^i}{S_t^i} = \gamma\sigma^2 dt + \sigma dZ_t + \sigma_1 dZ_t^i$$

■

- **COROLLARY 4:** The conditional expected jump in stock prices, as perceived just before time τ, is given by:

$$E_\tau[J_S] = -\frac{p_\tau(1-p_\tau)(1-F)(1-G)}{p_\tau + (1-p_\tau)FG} \qquad (A.12)$$

Proof of Corollary 5: At time τ, we have:

$$E_\tau[J_S] = p_\tau J_{S,\tau}^C + (1-p_\tau)J_{S,\tau}^I$$

$$E_\tau[J_S] = p_\tau \frac{(1-p_\tau)F(1-G)}{p_\tau + (1-p_\tau)FG} + (1-p_\tau)\frac{p_\tau(G-1)}{p_\tau + (1-p_\tau)FG}$$

$$E_\tau[J_S] = -\frac{p_\tau(1-p_\tau)(1-F)(1-G)}{p_\tau + (1-p_\tau)FG}$$

■

- **COROLLARY 5:** For $\sigma_C^2 = \sigma_I^2$, i.e., agents have the same level of uncertainty about partisanship effects, then $E_\tau[J_S] \geq 0$ for any combinations of μ_I and μ_C. Without any restrictions, $E_\tau[J_S] < 0$ if

$$\left(\gamma - \frac{1}{2}\right)(\sigma_I^2 - \sigma_C^2)(T-\tau) < \mu_I - \mu_C < \frac{\gamma}{2}(\sigma_I^2 - \sigma_C^2)(T-\tau) \qquad (A.13)$$

Proof of Corollary 5: Let's prove the second part of the corollary. Then, the first will be obvious. Given the equation for $E_\tau[J_S] = -\frac{p_\tau(1-p_\tau)(1-F)(1-G)}{p_\tau+(1-p_\tau)FG}$, $E_\tau[J_S] < 0$ if

$$-(1-F)(1-G) < 0 \Rightarrow (1-F)(1-G) > 0$$

$$\left(1 - e^{-\gamma(\mu_I-\mu_C)(T-\tau)+\frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)^2}\right)\left(1 - e^{(\mu_I-\mu_C)(T-\tau)+\left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)^2}\right) > 0$$

This is true if the product of the two exponents is positive:

$$\left(-\gamma(\mu_I-\mu_C) + \frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)\right)\left((\mu_I-\mu_C) + \left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)\right) > 0$$

$$\left((\mu_I-\mu_C) + \frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)\right)\left((\mu_I-\mu_C) + \left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)\right) < 0$$

This holds if:

$$\left(\gamma - \frac{1}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau) < \mu_C - \mu_I < \frac{\gamma}{2}(\sigma_I^2-\sigma_C^2)(T-\tau),$$

which proves the second part.

As shown before, $E_\tau[J_S] < 0$ if $\left((\mu_I-\mu_C) + \frac{\gamma^2}{2}(\sigma_I^2-\sigma_C^2)(T-\tau)\right)\left((\mu_I-\mu_C) + \left(\frac{1-2\gamma}{2}\right)(\sigma_I^2-\sigma_C^2)(T-\tau)\right) < 0$. Assuming that $\sigma_I^2 = \sigma_C^2$, we would have:

$$(\mu_I - \mu_C)^2 < 0\ ,$$

Which is never true. Thus, if $\sigma_I^2 = \sigma_C^2$, $E_\tau[J_S] \geq 0$.

■